

# Manifestations of Warped Extra Dimension in Rare Charm Decays and Asymmetries.

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## Abstract

Charm dynamics is moving back into focus with the established discovery of oscillations in the neutral  $D$  meson system and the sign of direct CP asymmetry in  $D^0 \rightarrow \pi^+\pi^-/K^+K^-$ . It opens the possibilities of finding CP violation beyond the reach of the Standard Model. In the recent past we have extensively studied charm dynamics within **non**-ad-hoc models with interesting flavour structures. We have shown that rare decays and CP violations are the best places to probe for New Dynamics in charm. We continue to study a different class of models, i.e., the Randall Sundrum model with a warped extras dimension to check for unusual effects in charm dynamics: namely in decays of final states with leptons and neutrinos and some asymmetries. These states should typically be accessible to experimental probes in the near future and, for certain, to any super flavour factory.

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Charm and the Standard Model</b>	<b>4</b>
2.1	$D^0 - \bar{D}^0$ : Experiments and Theory . . . . .	4
2.2	Rare decays of charm . . . . .	5
2.3	CP violation in $D^0 \rightarrow h^+ h^-$ . . . . .	8
<b>3</b>	<b>A Randall-Sundrum model with custodial isospin</b>	<b>9</b>
3.1	Gauge structure and electroweak symmetry breaking . . . . .	10
3.2	Quarks and leptons . . . . .	11
3.3	Couplings of the fermion and gauge mass eigenstates . . . . .	12
<b>4</b>	<b>The flavour structure and the parameter sets</b>	<b>14</b>
<b>5</b>	<b>Rare Decays</b>	<b>15</b>
5.1	$D^0 \rightarrow \gamma\gamma$ . . . . .	16
5.2	$D^0 \rightarrow \mu^+ \mu^-$ . . . . .	16
5.3	$D \rightarrow X_u \nu \bar{\nu}$ . . . . .	18
5.4	$D \rightarrow X_u l^+ l^-$ . . . . .	19
<b>6</b>	<b>Asymmetries in <math>D \rightarrow X_u l^+ l^-</math></b>	<b>20</b>
<b>7</b>	<b>Correlations between different <math>\Delta C = 1</math> observables</b>	<b>21</b>
<b>8</b>	<b>Correlations between strange, charm and beauty</b>	<b>23</b>
<b>9</b>	<b>Analysis of the parameter space</b>	<b>24</b>
<b>10</b>	<b>Inference</b>	<b>25</b>
<b>11</b>	<b>Summary</b>	<b>26</b>
<b>12</b>	<b>Acknowledgement</b>	<b>27</b>
	<b>References</b>	<b>27</b>

# 1 Introduction

One puzzle in high energy physics is the hierarchy that exists between the scale of electroweak symmetry breaking (EWSB) and the Planck scale,  $M_{Pl}$ , at which the effects of gravity must be taken into account. This Hierarchy Problem has inspired a great number of theories beyond the Standard Model (SM); one attractive solution being to assume the existence of a warped extra dimension. This class of models are known as Randall-Sundrum (RS) models [1]. They make use of the geometry of the extra dimension to provide a natural explanation for the hierarchy between the EWSB scale and  $M_{Pl}$ . The RS geometry is built from a slice of  $AdS_5$  bounded by two branes, where SM particles are allowed to propagate in the bulk between the electroweak or ‘infrared’ (IR) brane and the Planck or ‘ultraviolet’ (UV) brane. In order to properly address the Hierarchy Problem, the Higgs is localized near or at the IR brane and the size of the extra dimension is adjusted to properly redshift the Planck scale towards the electroweak scale. In addition to providing an attractive solution to the Hierarchy Challenge, different incarnations of the RS model have been implemented to address the fermion mass hierarchy [2–5]. This is possible by adjusting the localizations of the fermion fields relative to the Higgs five dimensional background profile. The existence of a warped extra dimension and the ability of matter to propagate in the bulk leads to the existence of an infinite tower of Kaluza-Klein (KK) modes in the four dimensional effective theory. The existence of KK modes can lead to sizable contributions to flavour changing neutral currents (FCNC) well beyond limits given by the data. Variations of the RS scenario have been constructed in order to suppress these contributions and remain consistent within current experimental constraints [6, 7].

The existence of electroweak precision data (EWPd) has placed very stringent bounds for the masses of KK modes. In particular, they constrain the masses of lowest KK mode excitations to  $M_{KK} \geq 12$  TeV for a strictly IR localized Higgs field [8] and  $M_{KK} \geq 7$  TeV for a Higgs field localized near the IR brane [9]. The LHC is on its way to probing large mass scales, but direct detection of new resonances will likely be possible close to the 1 TeV regime. This has motivated further research to extend the RS model in order to place KK excitations within the reach of the LHC. In particular, work by the authors of [10] makes use of an extended gauge group in the slice of  $AdS_5$ , protecting EW observables from very large contributions beyond experimental limits. They conclude that it is possible to fit EWPd with KK masses around 3 TeV. Furthermore, the authors in [11, 12] have argued that discovering these modes would be possible at the LHC with center of mass energies of 14 TeV and  $100 \text{ fb}^{-1}$  of integrated luminosities. Models that modify the  $AdS_5$  metric near the IR brane have also been shown to decrease the bounds on KK modes [9, 13–15]. Unlike the RS model, these extra dimensional structures are able to lower the KK mass scale to 2-3 TeV in large areas of their parameter space. Within this class of models, impact of KK modes involving third generation of quarks were shown to be within reach of the LHC with center of mass energy of 8 TeV and  $10 \text{ fb}^{-1}$  of integrated luminosities [16, 17].

Dynamics of charm – including its mere existence – has historically been the proving grounds for many subtle aspects of the Standard Model. In spite of that, charm has been for long been treated as the step child in flavour dynamics for the lack of experimental statistics and theoretical handle over its dynamics<sup>1</sup>. The recent evidence for direct CP violation in  $\Delta A_{CP}$  by the LHCb [18], the CDF [19, 20] and the Belle [21] collaborations has brought an end to those decades of neglect<sup>2</sup>. Although we do not address the implications of the class of models discussed in this work on the CP asymmetry in two-, three- and four- body hadronic decays of neutral charm mesons, and for very good reasons that we shall elucidate on later, we display other avenues through which warped geometry can leave its mark in charm dynamics. With the statistics available to the LHCb and in super flavour factories in the near future, these modes should be accessible through measurements of both decay rates and several asymmetries.

In this work, we study the effects of KK modes to rare decays of neutral  $D$  mesons within and RS framework with custodial isospin protection. As explained above, this class of models leads to KK mode excitations which are currently being probed by the LHC or will be in the near future. The structure of this model is a lot more complex, with the existence of an extended fermion sector. Furthermore, the presence of KK gauge bosons leads to FCNCs already at the tree level which has interesting implications for CP violating observables as well as rare decays of  $K$ ,  $B$  and  $D$  mesons. Signatures of this class of models along with the correlations generated by the same in  $K$  and  $B$  physics has been studied in great detail by [22–31]. They have extracted the flavour structure of these models as well as the fermion mass hierarchy which is consistent with current experimental bounds. Studies have also been done of the dependence of flavour dynamics on KK mass scales. In light of recent measurements by the

<sup>1</sup>“I know she invented fire – but what has she done recently?”

<sup>2</sup>Of course, we must not forget the “fans” of charm who supported it through out its not so glorious days and we pay them our due respect. CDF and Belle measurements are also sensitive to the individual asymmetries in  $D^0 \rightarrow K^+ K^-$  and  $D^0 \rightarrow \pi^+ \pi^-$ .

LHCb, CDF and Belle collaborations [18–21], we try to to compliment the work on  $K$  and  $B$  mesons combined with recent results on the  $B_s \rightarrow \mu^+ \mu^-$  branching fraction [32], and study the rare decays of neutral  $D$  mesons.

A new angle was introduced into our analysis by the very recent evidence of the time integrated branching fraction of  $B_s \rightarrow \mu^+ \mu^-$  at  $3.5\sigma$  significance [32]:

$$\text{BR}_{\text{exp}}(B_s \rightarrow \mu^+ \mu^-) = 3.2_{-1.2}^{+1.5} \times 10^{-9} . \quad (1)$$

It is quite consistent with the time integrated SM expectation of [33]

$$\text{BR}_{\text{SM}}(B_s \rightarrow \mu^+ \mu^-) = (3.54 \pm 0.30) \times 10^{-9} . \quad (2)$$

On the other hand this evidence leaves enough room for the intervention of new dynamics (ND) of several varieties, in this decay channel, it leaves us with the obvious doubt that SM might dominate over ND here and hence the latter might not be “found” here. We test our hypothesis of whether charm dynamics can allow significant ND intervention over SM effects even if the branching fraction of  $B_s \rightarrow \mu^+ \mu^-$  continues to look very akin to the SM expectation. While the SM produces the leading source of rare decays and asymmetries in  $B$  and  $K$  dynamics, the situation is very different for  $D$  transitions: SM gives tiny rates, and those are dominated by long distances dynamics. However, ND can enhance asymmetries of several classes significantly. Of course, one still has to deal with very small rates; however LHCb and super flavour factories will give us the needed huge data samples, while the SM might give little ‘backgrounds’.

The structure of this paper is as follows: In Section 2 we review the role of charm in the Standard Model and its current experimental status. We briefly discuss the different rare decay channels as well as the experimental evidence for CP violation in the  $D \rightarrow h^+ h^-$  channel at the Tevatron and the LHC. In Section 3, we introduce and review the main features of the RS model with custodial isospin protection. In Section 4, we discuss the flavour structure of the model along with the parameter space that we shall examine for  $\Delta C = 1$  processes. We calculate the new contributions to the effective Hamiltonian for the following  $\Delta C = 1$  observables in the decays:  $D^0 \rightarrow \mu^+ \mu^-$ ,  $D \rightarrow X_u \nu \bar{\nu}$  and  $D \rightarrow X_u l^+ l^-$  and discuss the results in Section 5. In Section 6 we look at the ND signature in several asymmetries in  $D \rightarrow X_u l^+ l^-$ . We discuss the correlations between several flavour observables in Sections 7 and 8. In Sections 9, 10 and 11 we analyse and summarise our results.

## 2 Charm and the Standard Model

In this section we build up a case for studying the effects of ND in charm dynamics. We go over the SM estimates in multiple channels and the experimental observables that have been under scrutiny and some more which can be scrutinized in the near future to give us some insight into the basic dynamics that affect charm hadrons.

### 2.1 $D^0 - \bar{D}^0$ : Experiments and Theory

That oscillations in a neutral charm meson system is a well established result now [34–36]. The HFAG average stands at [37]<sup>3</sup>:

$$x_D = \frac{\Delta M_D}{\Gamma_D} = (0.63_{-0.20}^{+0.19}) \% , \quad y_D = \frac{\Delta \Gamma_D}{2\Gamma_D} = (0.75 \pm 0.12) \%$$

$$\left| \frac{q}{p} \right| = 0.88_{-0.16}^{+0.18} , \quad \phi_D = (-10.1_{-8.9}^{+9.5})^\circ . \quad (3)$$

If it is assumed that there is no direct CP violation in charm dynamics, values of  $|q/p|$  and  $\phi_D$  come closer to their benchmark values for no CP violation:

$$\left| \frac{q}{p} \right| = 1.04_{-0.06}^{+0.07} , \quad \phi_D = (-2.02_{-2.74}^{+2.67})^\circ . \quad (4)$$

Recently, the LHCb Collaboration has presented measurements of oscillation parameters and ruled out the no oscillation hypothesis at  $9.1\sigma$  [38]. This is the first exclusion of the no oscillation hypothesis made from a single

<sup>3</sup>Up to date results can be found in the [Heavy Flavour Averaging Group's](#) website.

measurement. The measurement of the time dependent ratio between the doubly Cabbibo suppressed  $D^0 \rightarrow K^+ \pi^-$  to the Cabbibo allowed  $D^0 \rightarrow K^- \pi^+$  yields:

$$x'^2 = (-0.9 \pm 1.3) \times 10^{-4}, \quad y' = (7.2 \pm 2.4) \times 10^{-3}, \quad R_D = (3.5 \pm 0.15) \times 10^{-3}. \quad (5)$$

However, the relative size of  $x_D$  and  $y_D$  is not clear yet. *Before* these experimental results in 2007 [34–37], most authors had argued that the Standard Model (SM) predicts  $x_D, y_D \leq 3 \times 10^{-4}$  — yet not all. In 1998,  $x_D, y_D \leq 10^{-2}$  was listed, admittedly as a *conservative* SM bound [39], together with a question: How can one rule out that the SM can not produce  $10^{-6} \leq r_D \leq 10^{-4}$  (corresponding to  $x_D, y_D \sim 10^{-3} - 10^{-2}$ ). In 2000 and 2003, an SM prediction obtained from an operator product expansion (OPE) yielded  $x_D, y_D \sim \mathcal{O}(10^{-3})$  [40, 41]; later another OPE analysis was done with similar results [42, 43]. Alternatively in 2001 and 2004, an SM prediction on  $D^0 - \bar{D}^0$  oscillations was based on  $SU(3)$  breaking mostly in the phase space for  $y_D$  and then from a dispersion relation for  $x_D$  [44, 45].

The question of whether  $x_D$  and  $y_D$  can be accommodated within the SM is still open. Furthermore the situation is more complex: many theorists think that  $y_D$  depends almost all on long-distance dynamics and is hardly sensitive to impact from ND – in contrast to  $x_D$  that can be produced by ND (unless sources of ND suffer cancellations).

While hints of CP violation have been seen in recent experiments, it is still not clear whether these can be interpreted as a distinct signature of new dynamics (ND). However, there have been suggestions in the past that ND can possibly make large impact in charm dynamics [46–48]. The effect of models with a warped extra dimension in  $D^0 - \bar{D}^0$  oscillations has been considered in [28] and it was shown that certain parts of the parameter space can be constrained by the experimental limits in these observables available at that time.

## 2.2 Rare decays of charm

Rare charm decays within the SM have been studied extensively in the literature. The verdict is universal: SM leaves very small effects in these rare decays. Table 1 summarizes the SM effects in the rare decay channels that we will look into in this work. The results can be summarized as follows:

OBSERVABLE	SM SD	SM LD	EXPERIMENT
$\text{BR}(D^0 \rightarrow \gamma\gamma)$	$(3.6 - 8.1) \times 10^{-12} \dagger$	$(1 - 3) \times 10^{-8}$ [49, 50]	$< 2.4 \times 10^{-6}$ [51]
$\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$	$6 \times 10^{-19} \dagger$	$(2.7 - 8) \times 10^{-13}$	$< 1.3 \times 10^{-8}$
$\text{BR}(D \rightarrow X_u \nu \bar{\nu})$	$10^{-15} - 10^{-16} \dagger$	—	—
$\text{BR}(D^\pm \rightarrow X_u l^+ l^-)$	$3.7 \times 10^{-9} \dagger$	$\sim \mathcal{O}(10^{-6})$	$\sim \mathcal{O}(10^{-5})$
$A_{\text{FB}}^c$	$\sim 2 \times 10^{-6} \dagger$	—	—
$A_{\text{CP}}^c$	$\sim 3 \times 10^{-4} \dagger$	—	—
$A_{\text{FB}}^{\text{CP}}$	$\sim 3 \times 10^{-5} \dagger$	—	—

Table 1: SM short distance (SD) and long distance (LD) contributions to  $D^0 \rightarrow \gamma\gamma$ ,  $D^0 \rightarrow \mu^+ \mu^-$ ,  $D \rightarrow X_u \nu \bar{\nu}$  and  $D^\pm \rightarrow X_u l^+ l^-$ . Detailed calculations of the numbers marked with a  $(\dagger)$  can be found in the references [52–54].

### $D^0 \rightarrow \gamma\gamma$

The short distance contribution to this channel is about three orders of magnitude smaller than the long distance contribution. The long distance contribution comes primarily from a vector meson intermediate state and has been calculated in [49]. Alternatively, the long distance contribution has also been estimated using the hybrid chiral perturbation theory in [50]. New dynamics can possibly contribute to this channel

through short distance operators and hence has to enhance the short distance SM contribution by at least three orders of magnitude in order to overcome the SM LD contribution and make its presence felt. In our previous work on LHT-like models [52], we showed that new dynamics of this class can enhance the short distance contribution but not enough to overcome the long distance one.

Current experimental limits on this channel are quite far from the the SM estimates. Recent results from *BABAR* cite [51]:

$$\text{BR}_{\text{exp}}(D^0 \rightarrow \gamma\gamma) < 2.4 \times 10^{-6} \quad \text{at 90\% C.L.} \quad (6)$$

BESIII has updated their results recently [55, 56]. However, their limit falls short of the limit set by *BABAR*

$$\text{BR}_{\text{BESIII}}(D^0 \rightarrow \gamma\gamma) < 4.6 \times 10^{-6} \quad \text{at 90\% C.L.} \quad (7)$$

The preliminary BESIII result is with  $2.9\text{fb}^{-1}$  data at the  $\psi''(3770)$  peak. They have a projected reach of  $5 \times 10^{-8}$  with  $10\text{fb}^{-1}$  at the  $\psi''(3770)$  [55] peak within the next few years. A good measurement of the branching fraction of this channel will give us a very good estimate of the SM LD contribution to the branching fraction of  $D^0 \rightarrow \mu^+\mu^-$  as explained below.

### $D^0 \rightarrow \mu^+\mu^-$

The short distance contribution from SM to this channel is extremely tiny and is some six orders of magnitude smaller than the SM long distance contribution. The latter is driven by the two photon unitary contribution [49] and hence depends directly on the size of  $\text{BR}(D^0 \rightarrow \gamma\gamma)$  through the relation

$$\text{BR}_{\text{SM}}^{LD}(D^0 \rightarrow \mu^+\mu^-) \sim 2.7 \times 10^{-5} \text{BR}(D^0 \rightarrow \gamma\gamma). \quad (8)$$

New dynamics that can enhance  $\text{BR}(D^0 \rightarrow \gamma\gamma)$  above its SM value will indirectly enhance the SM LD contribution to  $\text{BR}(D^0 \rightarrow \mu^+\mu^-)$ . It is also believed that any new dynamics that contributes to  $D^0 - \bar{D}^0$  oscillations will also contribute to this decay channel significantly [57]. This is in general true. However, it should be kept in mind that such new dynamics can enhance the short distance SM contributions by many orders of magnitude but might fail to overcome the long distance contribution as we showed in [52]. The SM SD contribution to the branching fraction of  $D^0 \rightarrow \mu^+\mu^-$  is given by [58]:

$$\begin{aligned} \text{BR}(D^0 \rightarrow \mu^+\mu^-) = \frac{1}{\Gamma_{D^0}} \frac{G_F^2}{\pi} \left( \frac{\alpha}{4\pi \sin^2(\theta_W)} \right)^2 f_D^2 m_\mu^2 m_{D^0} \sqrt{1 - 4 \frac{m_\mu^2}{m_{D^0}^2}} \\ \times \sum_{j=d,s,b} |V_{cj}^* V_{uj}|^2 \left( Y_0(x_j) + \frac{\alpha_s}{4\pi} Y_1(x_j) \right)^2, \end{aligned} \quad (9)$$

where  $x_\mu = \mu^2/m_W^2$ . The definitions of  $Y_0(x_j)$  and  $Y_1(x_j)$  can be found in our previous work [52]. ND from the warped extra dimension can enter through tree level contributions to  $Y(x)$  as defined later in Section 5.

The LHCb now reports an upper bound of [59]:

$$\text{BR}_{\text{exp}}(D^0 \rightarrow \mu^+\mu^-) < 1.3 \times 10^{-8} \quad \text{at 95\% C.L..} \quad (10)$$

with  $0.9\text{fb}^{-1}$  of data. They can be expected to enhance this measurement by two or three orders of magnitude in the future. This measurement is out of reach for BESIII and can be within reach of super flavour factories.

### $D \rightarrow X_u \nu \bar{\nu}$

The SM contribution to this channel is extremely small. A naive estimate using the Inami-Lim functions [60] leads to the numbers [49]:

$$\text{BR}_{\text{SD}}(D^+ \rightarrow X_u \nu \bar{\nu}) \simeq 1.2 \times 10^{-15}, \quad \text{BR}_{\text{SD}}(D^0 \rightarrow X_u \nu \bar{\nu}) \simeq 5 \times 10^{-16}. \quad (11)$$

A complete analysis of SM contributions to this channel requires a full two-loop renormalization group analysis [61] as it is done for the light quark loops in the analogous  $K$  decay channel. This is necessitated by the fact that the quarks running in the loops of the penguin diagrams are light states and hence these operators cannot

be treated as purely short distance ones. Alternatively, these contributions can be treated as vector mesons in the intermediate state and such long distance contributions can be estimated [49]. The enhancement of such contributions over the SM SD contribution can be about an order of magnitude.

The impact of ND on this channel can be significant. In our analysis of this channel within the LHT-like framework [54] we showed that enhancements of  $O(10^3) - O(10^4)$  are possible. Considering the experimental sensitivity attainable in the immediate future, attaining theoretical precision in the calculation of the SM contribution to this decay channel is not a very fruitful enterprise. In spite of this, these channels are very demonstrative of the impact of ND over short distance SM dynamics and in the future super flavour factories, or in any future super charm factories, these decay channels with the requisite ND enhancement might open up for experimental probes.

### $D^\pm \rightarrow X_u l^+ l^-$

In both the inclusive and exclusive modes, the branching fraction is dominated by long distance effects driven by the  $\rho, \omega$  and  $\phi$  mesons. In size they are larger by two or three orders of magnitude than the quark level contribution to the inclusive mode, depending on the exclusive mode being looked into. The short distance contribution is primarily driven by the photon penguin and new dynamics of the LHT-like class can hardly make a dent there [53]. However, one can always look at the dilepton invariant mass distribution of the differential decay rate and distinguish between the short and the long distance contributions. Any deviation from the shape or size of this distribution as predicted by the standard model will be an unambiguous signal of new dynamics. The SM SD differential branching fraction for the inclusive process  $D \rightarrow X_u l^+ l^-$  is given by [53]:

$$\begin{aligned} \frac{d}{d\hat{s}} \text{BR}_{\text{SM}}^{\text{SD}}(D \rightarrow X_u l^+ l^-) = \\ \frac{1}{\Gamma_D} \frac{G_F^2 \alpha^2 m_c^5}{768 \pi^5} (1 - \hat{s})^2 \left[ \left( |C_9(\mu)|^2 + |C_{10}(\mu)|^2 \right) (1 + 2\hat{s}) + 12 \text{Re}(C_7(\mu) C_9^*(\mu)) + 4 \left( 1 + \frac{2}{\hat{s}} \right) |C_7(\mu)|^2 \right], \end{aligned} \quad (12)$$

with

$$\hat{s} = \frac{(p_{l^+} + p_{l^-})^2}{m_c^2}.$$

We set  $\mu = m_c = 1.2$  GeV. Integrating over  $\hat{s}$  gives us the total decay rate. One has to be careful about not picking up the infrared divergence in the differential decay rate. We made an infrared cut on  $\hat{s}$  at about an invariant dilepton momentum of 20 MeV. The definitions of the Wilson operators and the form of the coefficients can be found in [53]. Models with a warped extra dimension can leave their impact in  $C_9$  and  $C_{10}$  through tree level contributions to  $Y(x)$  and  $Z(x)$  defined below in Section 5.

### Asymmetries in $D^\pm \rightarrow X_u l^+ l^-$

Although the branching ratios are dominated by long distance physics, the asymmetries are sensitive to mostly short distance physics. We clearly showed in [53] that if one kinematically cuts out the vector resonances in the invariant dileptonic mass distribution, one loses effects of  $O(10\%)$  only in the asymmetries. Hence asymmetries are good observables for the discovery of ND as the latter is expected to bring enhancements to the short distance dynamics. There are three asymmetries that can be probed in this channel which come solely from  $\Delta C = 1$  currents [53].

The forward-backward asymmetry  $A_{\text{FB}}^c$  is tiny in the SM,  $O(10^{-6})$ , and the impact of new dynamics here can be quite large. While this held true in the LHT-like model we looked at in our previous work [53], it was also true that the absolute size of the asymmetry remained at  $O(1\%)$ . The normalized forward-backward asymmetry is defined from the double differential decay rate as

$$A_{\text{FB}}^c(\hat{s}) = \frac{\int_{-1}^1 \left[ \frac{d^2}{d\hat{s} dz} \Gamma(D^\pm \rightarrow X_u l^+ l^-) \right] \text{sgn}(z) dz}{\int_{-1}^1 \left[ \frac{d^2}{d\hat{s} dz} \Gamma(D^\pm \rightarrow X_u l^+ l^-) \right] dz}. \quad (13)$$

After performing the integral over the angular distribution we obtain

$$A_{\text{FB}}^c(\hat{s}) = \frac{-3 [\Re(C_{10}^*(\mu)C_9(\mu))\hat{s} + 2\Re(C_{10}^*(\mu)C_7(\mu))]}{(1 + 2\hat{s}) \left( |C_9(\mu)|^2 + |C_{10}(\mu)|^2 \right) + 4|C_7(\mu)|^2 \left( 1 + \frac{2}{\hat{s}} \right) + 12\Re(C_7(\mu)C_9^*(\mu))}. \quad (14)$$

Enhancements to both  $C_9$  and  $C_{10}$  can manifest themselves as sizable  $A_{\text{FB}}^c$ .

The CP asymmetry  $A_{\text{CP}}^c$  is of  $O(10^{-4})$  in the SM and new dynamics like the LHT-like models can hardly leave much dent there. This can be well understood in terms of the absence of tree level contribution to the CP asymmetry in these class of models. However, this is not true for the models with a warped extra dimension and hence we check for enhancement to the CP asymmetry. The CP asymmetry parameter  $A_{\text{CP}}^c(\hat{s})$  is defined as

$$A_{\text{CP}}^c(\hat{s}) = \frac{\frac{d}{d\hat{s}}\Gamma(D^+ \rightarrow X_u l^+ l^-) - \frac{d}{d\hat{s}}\Gamma(D^- \rightarrow X_{\bar{u}} l^+ l^-)}{\frac{d}{d\hat{s}}\Gamma(D^+ \rightarrow X_u l^+ l^-) + \frac{d}{d\hat{s}}\Gamma(D^- \rightarrow X_{\bar{u}} l^+ l^-)}. \quad (15)$$

Integrating over the invariant dileptonic mass we get the total CP asymmetry  $A_{\text{CP}}^c$

The CP asymmetry in the forward-backward asymmetry  $A_{\text{FB}}^{\text{CP}}$  can show very large contributions from new dynamics like the LHT. The SM contribution to this asymmetry is of  $O(10^{-5})$ . New dynamics can enhance this by an order of  $O(10\% - 100\%)$  [53]. Since this asymmetry is sensitive to phases in the Wilson coefficients, it is open to enhancements by ND, and there is a good possibility that models with a warped extra dimension can enhance this asymmetry to quite large values. The normalized difference in the forward-backward asymmetry in  $D \rightarrow X_u l^+ l^-$  and  $\bar{D} \rightarrow X_{\bar{u}} l^+ l^-$  is defined as [62]

$$A_{\text{FB}}^{\text{CP}}(\hat{s}) = \frac{A_{\text{FB}}^c(\hat{s}) + A_{\text{FB}}^{\bar{c}}(\hat{s})}{A_{\text{FB}}^c(\hat{s}) - A_{\text{FB}}^{\bar{c}}(\hat{s})}. \quad (16)$$

In the limit of CP symmetry,  $A_{\text{FB}}^c(\hat{s})$  and  $A_{\text{FB}}^{\bar{c}}(\hat{s})$  have to be exactly equal in magnitude but with an opposite sign [63, 64]. As the forward-backward asymmetry is defined in terms of the positive anti-lepton,  $A_{\text{FB}}^c(\hat{s})$  and  $A_{\text{FB}}^{\bar{c}}(\hat{s})$  have opposite signs.  $A_{\text{FB}}^{\text{CP}}(\hat{s})$  is sensitive to the phase in  $C_7$ ,  $C_9$  and  $C_{10}$ . The SM offers phases only in  $C_7$  and  $C_9$  in  $D \rightarrow X_u l^+ l^-$  and none in  $C_{10}$ . Hence the integrated asymmetry turns out to be very small.

$$\int A_{\text{FB}}^{\text{CP}}(\hat{s}) d\hat{s} = A_{\text{FB}}^{\text{CP}} \sim 3 \times 10^{-5}. \quad (17)$$

To make the study of these asymmetries “clean” both theoretically and experimentally, cuts in the dilepton mass distribution can be made around the  $\rho$ ,  $\omega$  and  $\phi$  resonances. Our previous study showed that making such cuts left our results unaltered proving that long distance dynamics, especially the resonances, have very little to do in these asymmetries. It should also be emphasized that the procedure of kinematically “cutting” out the resonances can be done both in theory and experiments leaving equivalent impacts on the observables.

### 2.3 CP violation in $D^0 \rightarrow h^+ h^-$

There are two interesting results for CP violation in  $D^0 \rightarrow \pi^+ \pi^- / K^+ K^-$ , both from hadronic experiments. The CDF results can be summarized as [19]:

$$\begin{aligned} \langle A_{\text{CP}}^{\text{CDF}}(D^0 \rightarrow \pi^+ \pi^-) \rangle &= (+0.22 \pm 0.24_{\text{stat}} \pm 0.11_{\text{syst}})\%, \\ \langle A_{\text{CP}}^{\text{CDF}}(D^0 \rightarrow K^+ K^-) \rangle &= (-0.24 \pm 0.22_{\text{stat}} \pm 0.10_{\text{syst}})\%. \end{aligned} \quad (18)$$

The result from LHCb [18] is cast in a different form to cancel inherent production asymmetries:

$$\Delta A_{\text{CP}}^{\text{LHCb}} = A_{\text{CP}}(D^0 \rightarrow K^+ K^-) - A_{\text{CP}}(D^0 \rightarrow \pi^+ \pi^-) = [-0.82 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})]\%. \quad (19)$$

CDF also quotes  $\Delta A_{\text{CP}}$  [20]:

$$\Delta A_{\text{CP}}^{\text{CDF}} = [-0.62 \pm 0.21(\text{stat.}) \pm 0.10(\text{syst.})]\%. \quad (20)$$



Of course, we should not overlook the recent Belle result using  $976\text{fb}^{-1}$  of data [21]:

$$\begin{aligned}\langle A_{\text{CP}}^{\text{Belle}}(D^0 \rightarrow \pi^+\pi^-) \rangle &= (+0.55 \pm 0.36_{\text{stat}} \pm 0.09_{\text{syst}})\% , \\ \langle A_{\text{CP}}^{\text{Belle}}(D^0 \rightarrow K^+K^-) \rangle &= (-0.32 \pm 0.21_{\text{stat}} \pm 0.09_{\text{syst}})\% ,\end{aligned}\tag{21}$$

$$\Delta A_{\text{CP}}^{\text{Belle}} = [-0.87 \pm 0.41(\text{stat.}) \pm 0.06(\text{syst.})]\% .\tag{22}$$

The CDF and Belle results are sensitive to both direct and indirect CP violation. However, the difference in the CP asymmetry parameters,  $\Delta A_{CP}$  measured by all three experiments is sensitive mostly to *direct* CP violation. This holds true as it is well agreed upon that indirect CP violation has to be universal and hence mostly final state independent, which leads to a cancellation when differences in CP asymmetry in two modes are taken. It should be kept in mind though that  $\Delta A_{CP}$  does contain a part of the indirect CP violation through the interference of mixing with direct CP violation.

A second subtlety that should be kept in mind is that it is generally agreed upon that the sign of the direct CP asymmetry in the  $K^+K^-$  decay mode is opposite to that in the  $\pi^+\pi^-$  decay mode. However, the sizes of the asymmetry need not be related and are equal only in the limit of exact  $SU(3)_F$  symmetry.<sup>4</sup> This means that the asymmetries really add up in the difference.

For indirect CP violation, significant deviation of the observed values of  $|q/p|$  from 1 and  $\phi_D$  from 0 or  $\pi$  opens up possibilities of large CP violation. On the other hand, tiny values for  $x_D$  and  $y_D$  effectively suppress these effects. Hence, the fact that indirect CP violation is expected to be small in charm mesons is a consequence of the size of the oscillation parameters and is not related to the phase contributions to CP violation *per se*.

Within the SM, it is believed that direct CP violation in these modes should be of  $O(10^{-4})$  and indirect CP violation should be of  $O(10^{-5})$  [66–69]. Direct CP violation of  $O(10^{-3})$ , however, cannot be claimed as an unambiguous signal of new dynamics. For that claim to hold, direct CP violation of  $O(10^{-2})$  has to be observed. In quite a few scenarios of new dynamics indirect CP violation can be enhanced without enhancing direct CP violation much above what the SM has to offer. Motivated by the initial results of CDF, we had shown that scenarios like the LHT cannot enhance direct CP violation while it can saturate the limits of indirect CP violation [66]. There are some others in which both can be enhanced. A lot of work has been come forward in the literature since the announcement of the LHCb results showing how direct CPV can be significantly enhanced by different scenarios of ND [6, 70–81]. However, it can be equally well argued that the SM can accommodate the present experimental evidence [70, 82–85]. Considering the uncertainties shrouding the SM estimate of CPV in this channel and moreover the conceptual shortcomings of the theoretical technologies used to make these estimate, it is quite impossible to make a simple statement of whether the measured value of  $\Delta A_{CP}$  is within the SM reach or is the signature of ND [70, 71, 82, 86].

The work which is most closely related to the kind of models we will study in this work is reference [77], where an estimate is made on how the localisation of the Higgs in the bulk impacts the direct CP asymmetry in  $D^0 \rightarrow \pi^+\pi^-/K^+K^-$ . In the light of the conceptual uncertainties plaguing the precise determination of both the SM and the ND contribution to the direct CP asymmetry in hadronic decays of charm, we shall refrain from making an estimate of the same. Suffice it to mention here that it can be inferred that models of this kind can well produce the CP asymmetry under question. Whether that is what is actually happening can only be answered if and when more rigorous theoretical tools have been developed and/or experiments definitively make a case for SM or ND.

### 3 A Randall-Sundrum model with custodial isospin

In this section we review the RS model with custodial isospin protection [10]. We examine how electro weak symmetry breaking (EWSB) proceeds and its implication for the spectrum, in particular how it leads to FCNC at tree level. Additionally, we review the larger fermion structure of this model and how it leads to fermion mass mixing beyond the CKM structure.

<sup>4</sup>More specifically, the asymmetries are equal in magnitude and have opposite sign in the limit of exact U-spin symmetry. The implications of U-spin was first pointed out in the context of DCS decays Gronau and Rosner in [65] and later in the context of CP violation in SCS decays by Grossman et. al. in [69]. We are assuming here that isospin symmetry holds good.

### 3.1 Gauge structure and electroweak symmetry breaking

The RS model with custodial isospin consists of an extended gauge symmetry in the bulk given by:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}. \quad (23)$$

The model is constructed in a slice of  $AdS_5$  described by the following metric:

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2. \quad (24)$$

The UV brane is located at  $L = 0$  and the IR brane at a distance  $y = L$  along the extra dimension. In the bulk, the Lagrangian for the gauge sector is given by

$$L_{gauge} = \sqrt{G} \left( \frac{1}{4} Tr W_{MN} W^{MN} - \frac{1}{4} Tr \tilde{W}_{MN} \tilde{W}^{MN} - \frac{1}{4} \tilde{B}_{MN} \tilde{B}^{MN} - \frac{1}{4} Tr G_{MN} G^{MN} \right), \quad (25)$$

where  $W_{MN}$  is the field strength for the  $SU(2)_L$  gauge group,  $\tilde{W}_{MN}$  for  $SU(2)_R$ , and  $\tilde{B}_{MN}$  and  $G_{MN}$  for  $U(1)_X$  and QCD gauge groups respectively. By assigning the appropriate boundary conditions, (UV,IR), one is able to recover the electroweak (EW) gauge symmetry in the UV brane:

$$\begin{aligned} W_\mu^{1,2,3}(++), \quad \tilde{B}_\mu(++), \\ \tilde{W}_\mu^{1,2}(-+), \quad \tilde{W}_\mu^3(++), \end{aligned} \quad (26)$$

where  $(+)/(-)$  denote Neumann/Dirichlet boundary conditions. Furthermore, breaking of the bulk symmetry in the UV brane down to  $SU(2)_L \times U(1)_Y$  leads to mixing between  $\tilde{B}_\mu$  and  $\tilde{W}_\mu^3$ :

$$\begin{aligned} Z'_\mu &= \tilde{W}_\mu^3 \cos \phi - \tilde{B}_\mu \sin \phi, \\ B_\mu &= \tilde{W}_\mu^3 \sin \phi + \tilde{B}_\mu \cos \phi, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \cos \phi &= \frac{\tilde{g}_5}{\sqrt{\tilde{g}_5^2 + \tilde{g}'_5{}^2}}, \\ \sin \phi &= \frac{\tilde{g}'_5}{\sqrt{\tilde{g}_5^2 + \tilde{g}'_5{}^2}}. \end{aligned} \quad (28)$$

This symmetry breaking pattern guarantees massless zero modes for the fields with  $(+,+)$  boundary conditions.

Electroweak symmetry breaking is achieved as in the RS model [1], introducing a scalar Higgs field, which is now localized near the IR brane. This field which transforms as a bidoublet of  $SU(2)_R \times SU(2)_L$  is given by

$$\Sigma = \begin{pmatrix} \frac{\pi^+}{\sqrt{2}} & -\frac{h^0 - i\pi^2}{2} \\ \frac{h^0 + i\pi^2}{2} & \frac{\pi^-}{\sqrt{2}} \end{pmatrix} \quad (29)$$

The neutral component of the Higgs field acquires a  $vev$  which breaks the EW bulk symmetry,  $SU(2)_R \times SU(2)_L$ , down to its diagonal combination  $SU(2)$ , leading to an unbroken custodial symmetry. As can be seen from the pattern of EWSB in the UV as well as the IR branes, the discrete symmetry,  $P_{LR}$ , remains unbroken, essentially protecting non-oblique parameters from large contributions beyond experimental limits.

After EWSB, the photon and gluon zero modes remain massless. Zero modes of fields coupling to the Higgs,  $W_\mu^{1,2,3}$ , acquire a mass and the following mixing pattern arises between the neutral components of the gauge fields:

$$\begin{aligned} Z_\mu &= W_\mu^3 \cos \psi - B_\mu \sin \psi, \\ A_\mu &= W_\mu^3 \sin \psi + B_\mu \cos \psi. \end{aligned} \quad (30)$$

The mixing angle  $\psi$  can be expressed in terms of the angle  $\phi$  using the following relations:

$$\begin{aligned}\cos \psi &= \frac{1}{\sqrt{1 + \sin^2 \phi}}, \\ \sin \psi &= \frac{\cos \phi}{\sqrt{1 + \sin^2 \phi}}.\end{aligned}\tag{31}$$

Within this framework the physical  $Z$  mass is generated through mixing between the zero and first<sup>5</sup> KK modes of  $Z_\mu$  with  $Z'_\mu$ ; similarly for the charged gauge bosons.

As in the RS model, profiles for the gauge boson KK modes are given by

$$f^n(y) = \frac{e^{ky}}{N_n} \left( J_1 \left( \frac{m_n}{k} e^{ky} \right) + b_1(m_n) Y_1 \left( \frac{m_n}{k} e^{ky} \right) \right), \tag{32}$$

where  $J_1$  and  $Y_1$  are Bessel functions of the first and second kind,  $m_n$  is the  $n^{th}$  order KK mode mass, and  $N_n$  is a normalization factor.

### 3.2 Quarks and leptons

Fermions in this model have to be embedded in representations of the bulk symmetry since they propagate in the bulk. Additionally, contributions to EWPD beyond experimental limits need to be suppressed, such as  $Zb\bar{b}$ . This can be done by embedding fermions within representations of the group  $O(4)$ , which is isomorphic to  $SU(2)_L \times SU(2)_R \times P_{LR}$  [87]. As a consequence, this embedding will yield a larger fermion structure.

In particular, left handed up- and *down* type quarks are embedded within bidoublet representations of  $S(2)_L \times SU(2)_R$ , with the right handed *up* type quarks transforming as singlets under  $O(4)$ . The right handed *down* type quarks transform as a  $(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3})$ . As with gauge bosons, fields with Neumann boundary conditions at both branes,  $(++)$ , give rise to massless zero modes, leading to the SM model spectrum. The electric charges are given by

$$Q = T_L^3 + T_R^3 + Q_X, \tag{33}$$

where  $T_{L,R}^3$  denote the third component of  $SU(2)_{L,R}$  and  $Q_X$  the  $U(1)_X$  charge. Through this framework, all fermions receive the same  $U(1)_X$  charge and after electroweak symmetry breaking, mixing occurs between fermions with the same electric charge. A detailed study of the mixing between zero modes and the lowest KK modes is carried out in [22]. Here we just summarize the most important results that lead to the quark mass matrices. The structure of the model consists of three mass matrices for the states with charges 5/3, 2/3, and  $-1/3$ . For states with left handed chirality, these can be grouped into the following vectors:

$$\begin{aligned}\Psi_L(5/3)^T &= (\chi_L^{u_i}(-+), \psi_L^{i_i}(+-), \psi_L^{i_i}(-+))_{Q_X=2/3}, \\ \Psi_L(2/3)^T &= (q_L^{u_i(0)}(++), q_L^{u_i}(++), U_L^{i_i}(+-), U_L^{i_i}(-+), \chi_L^{d_i}(-+), u_L^{i_i}(--))_{Q_X=2/3}, \\ \Psi_L(-1/3)^T &= (q_L^{d_i(0)}(++), q_L^{d_i}(++), D_L^{i_i}(+-), D_L^{i_i}(--))_{Q_X=2/3}.\end{aligned}\tag{34}$$

Similarly, right handed fermions are obtained by replacing  $L \rightarrow R$  in the above expressions.

The structure of the Yukawa interactions have to preserve the  $O(4)$  symmetry. This leads to the following four dimensional Yukawa matrices:

$$\begin{aligned}\left(Y_{ij}^{(5/3)}\right)_{kl} &= \frac{1}{\sqrt{2}L^{3/2}} \int_0^L dy \lambda_{ij}^d f_{L,k}^{(5/3)}(y) f_{R,l}^{(5/3)}(y) h(y), \\ \left(Y_{ij}^{(2/3)}\right)_{kl} &= \frac{1}{\sqrt{2}L^{3/2}} \int_0^L dy \lambda_{ij}^d f_{L,k}^{(2/3)}(y) f_{R,l}^{(2/3)}(y) h(y), \\ \left(\tilde{Y}_{ij}^{(2/3)}\right)_{kl} &= \frac{1}{\sqrt{2}L^{3/2}} \int_0^L dy \lambda_{ij}^u f_{L,k}^{(2/3)}(y) f_{R,l}^{(2/3)}(y) h(y), \\ \left(Y_{ij}^{(-1/3)}\right)_{kl} &= \frac{1}{\sqrt{2}L^{3/2}} \int_0^L dy \lambda_{ij}^d f_{L,k}^{(-1/3)}(y) f_{R,l}^{(-1/3)}(y) h(y),\end{aligned}\tag{35}$$

<sup>5</sup>We do not consider the higher KK modes as they have progressively smaller impacts.

where the functions  $f_{L,R}$  are the fermion 5D profiles given by [2, 4]

$$f_{L,R}^0(y) = \sqrt{\frac{(1 \mp 2c)kL}{e^{(1 \mp 2c)kL} - 1}} e^{\mp cky}, \quad (36)$$

for the zeroeth mode and

$$f_{L,R}^n(y) = \frac{e^{ky/2}}{N_n} \left[ J_\alpha \left( \frac{m_n}{k} e^{ky} \right) + b_\alpha(m_n) Y_\alpha \left( \frac{m_n}{k} e^{ky} \right) \right], \quad (37)$$

with  $\alpha = |\pm c + 1/2|$  for the  $n^{th}$  KK mode.  $\lambda^{u,d}$  denote the fundamental 5D Yukawa couplings and  $c$  the bulk mass of the 5D fermion field. The Higgs field background profile is given by the function  $h(y)$ . Using the weak eigenstates in Equation (34) and the effective 4D Yukawa couplings in Equation (35) one can write the following Lagrangian which mixes zero and first KK modes with the same electric charge [22]

$$\begin{aligned} \mathcal{L} = & -\bar{\Psi}_L(5/3)\mathcal{M}(5/3)\Psi_R(5/3) + \text{h.c.} \\ & -\bar{\Psi}_L(2/3)\mathcal{M}(2/3)\Psi_R(2/3) + \text{h.c.} \\ & -\bar{\Psi}_L(-1/3)\mathcal{M}(-1/3)\Psi_R(-1/3) + \text{h.c.} \end{aligned} \quad (38)$$

The lepton sector can be accommodated in a similar fashion, but with the appropriate choice of  $U(1)_X$  charge,  $Q_X \rightarrow 0$ . Within this model, left handed charged leptons and neutrinos are grouped within a bidoublet of  $SU(2)_L \times SU(2)_R$ , and the right handed charged leptons transform as a  $(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3})$  of the EW symmetry. In addition, this model contains a right handed neutrino field transforming as a singlet of the  $O(4) \times U(1)_X$  bulk symmetry.

### 3.3 Couplings of the fermion and gauge mass eigenstates

To calculate the tree level contributions to FCNC's we first derive the couplings of fermions to the electroweak neutral gauge bosons. Because of the mixing that arises between the zero and first KK modes of  $Z_\mu$  with  $Z'_\mu$ , we must diagonalize the mass terms in order to find the appropriate mass eigenstates. In this section we will work in the EW basis for fermions. The rotation to mass eigenstates will be shown in the following section, where the CKM structure of the model will be discussed in greater depth. In order to derive the couplings between neutral gauge bosons and  $up$  type fermions we follow the analysis in [87]. The left handed  $up$  type quarks transform as a bidoublet of the full  $O(4) \times U(1)_X$  symmetry and couple to the gauge bosons through the following Lagrangian:

$$\mathcal{L} \supset c_1 \text{Tr}[\bar{Q}_L \gamma^\mu Q_L \hat{V}_\mu] + c_2 \text{Tr}[\bar{Q}_L \gamma^\mu V_\mu Q_L] + (c_3 \text{Tr}[\bar{Q}_L \gamma^\mu i D_\mu U] \text{Tr}[U^\dagger Q_L] + \text{h.c.}), \quad (39)$$

where  $Q_L \in (\mathbf{2}, \mathbf{2})_{2/3}$  of  $O(4) \times U(1)_X$ ,  $V_\mu = (i D_\mu U) U^\dagger$ , and  $\hat{V}_\mu = (i D_\mu U)^\dagger U$ . We make use of the following covariant derivative

$$D_\mu U = \partial_\mu U + i \frac{g_5}{2} \sigma_a W_\mu^a U + i \frac{\tilde{g}_5}{2} \sigma_a \tilde{W}_\mu^a U - i \frac{\tilde{g}_5'}{2} B_\mu U \sigma_3, \quad (40)$$

where  $U$  is a non-linear sigma field which contains the pseudo Nambu-Goldstone bosons resulting from the breaking of  $O(4)$  into  $O(3)$ . The  $P_{LR}$  symmetry of the model leads to the conditions  $g_5 = \tilde{g}_5$  and  $c_1 = c_2$ ; guaranteeing the vanishing of a potentially large contribution to  $Z b_L \bar{b}_L$  coupling which would violate experimental limits. We have naively used effective four dimensional fields together with the fundamental 5D gauge coupling. To obtain the effective 4D coupling, integration over the fermion and gauge boson profiles must be carried out with  $U = \mathbb{1}_{2 \times 2}$  after the  $O(4)$  symmetry is broken. Contributions to the right handed  $up$  type quarks vanish, since these transform as singlets under the  $O(4)$  symmetry, coupling in the usual sense to the  $U(1)_X$  gauge boson.

The couplings of left handed  $up$  type quarks to the  $Z^0$ ,  $Z^1$  and  $Z'$  are given by

$$\begin{aligned} g_{L,Z^0} &= \frac{g_4}{\cos \psi} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \psi \right) F_L^0(c_{L,i}, c_{L,i}), \\ g_{L,Z^1} &= \frac{g_4}{\cos \psi} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \psi \right) F_L^1(c_{L,i}, c_{L,i}), \\ g_{L,Z'} &= \frac{g_4}{\cos \phi} \left( -\frac{1}{2} - \frac{1}{6} \sin^2 \phi \right) F_L'(c_{L,i}, c_{L,i}), \end{aligned} \quad (41)$$

and those of right handed  $up$  type quarks by

$$\begin{aligned} g_{R,Z^0} &= -\frac{2}{3} \frac{g_4}{\cos \psi} \sin^2 \psi F_R^0(c_{R,i}, c_{R,i}), \\ g_{R,Z^1} &= -\frac{2}{3} \frac{g_4}{\cos \psi} \sin^2 \psi F_R^1(c_{R,i}, c_{R,i}), \\ g_{R,Z'} &= -\frac{2}{3} \frac{g_4}{\cos \phi} \sin^2 \phi F_R'(c_{R,i}, c_{R,i}), \end{aligned} \quad (42)$$

where  $g_4$  denotes the  $SU(2)_L$  coupling strength as in the SM model, and  $F_{L,R}^{0,1}(c_{L,R,i}, c_{L,R,i})$  are diagonal matrices that quantify the overlap between right and left handed zero-mode fermions with the  $Z^{0,1}$  gauge bosons. These matrices give rise to FCNC's at the tree level given that they are not proportional to the identity. The coupling to the photon is given by

$$g_{L,R} = \frac{2}{3} g_4 \sin \psi. \quad (43)$$

The overlap is identical to that of the  $Z^1$  gauge boson since at this stage they both have the same 5D profile. The effects of EWSB arise at the mixing level between the three neutral gauge bosons. The mixing between the neutral gauge bosons can be parametrized by a unitary matrix  $GZ$  used to diagonalize the masses of the neutral gauge bosons [22]. Therefore, the mass eigenstates can be ordered in the following way:

$$(Z^0, Z^1, Z')^T = GZ(Z_3, Z_2, Z_1)^T, \quad (44)$$

where  $Z_1$  denotes the lightest eigenstate and the couplings in Equations (41) and (42) transform such that  $g_{L,R}(i) \rightarrow g_{L,R}(i) GZ_{ij}$ .

The analysis of the interactions between neutral gauge bosons and leptons is carried out in the same way. The interactions of the left handed leptons are obtained from Equation (39), which leads to the following effective couplings:

$$\begin{aligned} g_{L,Z^0} &= -\frac{g_4}{\cos \psi} \left( \frac{1}{2} \sin^2 \psi \right) F_L^0(c_{L,i}, c_{L,i}), \\ g_{L,Z^1} &= -\frac{g_4}{\cos \psi} \left( \frac{1}{2} \sin^2 \psi \right) F_L^1(c_{L,i}, c_{L,i}), \\ g_{L,Z'} &= \frac{g_4}{\cos \phi} \left( -\frac{1}{2} + \frac{1}{2} \sin^2 \phi \right) F_L'(c_{L,i}, c_{L,i}). \end{aligned} \quad (45)$$

Right handed leptons belong to the  $(\mathbf{1}, \mathbf{3})$  triplet representation of  $SU(2)_L \times SU(2)_R$ . An effective Lagrangian analysis yields two additional contributions to their couplings to neutral gauge bosons [87]. These can be parametrized by the following effective Lagrangian

$$\mathcal{L} \supset c_4 Tr[\bar{E}_R \gamma^\mu E_R \hat{V}_\mu] + c_5 Tr[\bar{E}_R \gamma^\mu V_\mu E_R]. \quad (46)$$

The effective couplings are given by:

$$\begin{aligned} g_{R,Z^0} &= \frac{g_4}{\cos \psi} \sin^2 \psi F_R^0(c_{R,i}, c_{R,i}), \\ g_{R,Z^1} &= \frac{g_4}{\cos \psi} \sin^2 \psi F_R^1(c_{R,i}, c_{R,i}), \\ g_{R,Z'} &= -g_4 \cos \phi F_R'(c_{R,i}, c_{R,i}), \end{aligned} \quad (47)$$

while those to the photon KK mode are given by  $g_{L,R} = -g_4 \sin \psi$ .

In Section 5 we show the new tree-level contributions to the rare decays of  $D$  mesons due to the couplings introduced above using a KK mass scale of 2.45 TeV. Before proceeding, in Section 4 we discuss the flavour structure of the model and the parameter sets used in the analysis of  $D$  meson rare decays.

## 4 The flavour structure and the parameter sets

The flavour structure of this model was studied in depth in [22, 24]. Alternative approaches can be found in [28, 88]. Before looking at the structure of the CKM matrix, it is worth emphasizing that the mixing of zero modes with heavier KK modes, Equation (38), leads also to a source of FCNC. However as it was pointed out in [24] as well as [6, 88], this effects are negligible and thus we only consider the  $k = l = 0$  components of the 4D Yukawa matrices, Equation (35), when rotating to the mass eigenstate basis. The 4D Yukawas can then be effectively written as

$$Y_{ij}^{u,d} \equiv \lambda_{ij}^{u,d} \frac{e^{kL}}{kL} f_L^0(y=L, c_Q^i) f_R^0(y=L, c_{u,d}^j), \quad (48)$$

where  $\lambda_{ij}^{u,d}$  denote the 5D Yukawa couplings. The CKM matrix is obtained as in the SM, that is

$$V_{CKM} = U_L^\dagger D_L, \quad (49)$$

where  $U_L$  and  $D_L$  are unitary matrices which rotate flavour eigenstates into mass eigenstates for left handed up- and *down* type quarks. Their complete parametrization can be found in [22, 24]. Since we are considering neutral current exchanges at tree level, we found it useful to define effective unitary matrices for the *up* type quarks,  $U_{L,eff}$  and  $U_{R,eff}$  coupling quark mass eigenstates to the three physical massive neutral gauge bosons as well as to the first KK mode of the photon. These can be obtained using Equations (41), (42), and (43):

$$\begin{aligned} U_{L,eff}(n) &= g_{L,Z^0} \cdot GZ_{1n} + U_L^\dagger g_{L,Z^1} U_L \cdot GZ_{2n} + U_L^\dagger g_{L,Z'} U_L \cdot GZ_{3n}, \\ U_{R,eff}(n) &= g_{R,Z^0} \cdot GZ_{1n} + U_R^\dagger g_{R,Z^1} U_R \cdot GZ_{2n} + U_R^\dagger g_{R,Z'} U_R \cdot GZ_{3n}, \end{aligned} \quad (50)$$

for the three physical neutral gauge bosons denoted by  $n = 1, 2, 3$  and

$$\begin{aligned} U_{L,eff}(\gamma') &= U_L^\dagger g_L U_L, \\ U_{R,eff}(\gamma') &= U_R^\dagger g_R U_R, \end{aligned} \quad (51)$$

for the first KK mode of the photon. In what follows we describe how the parameters encoded in the matrices  $g_{L,R}$  are constrained, in particular, their dependence on the bulk mass parameter.

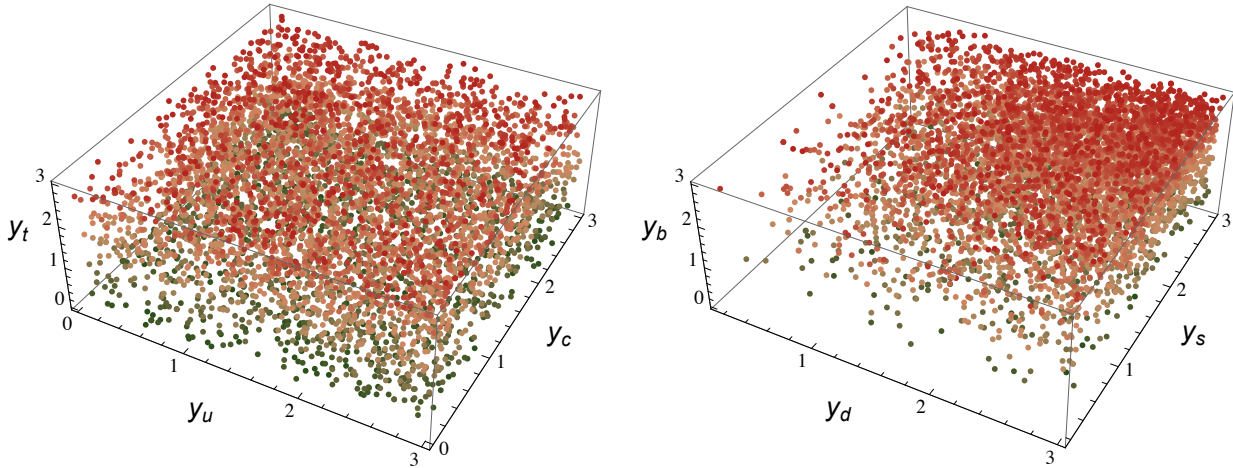


Figure 1: Scatter plot of the Yukawa couplings of the quarks. The figure on the left is for the *up* type quarks and that on the right is for the *down* type quarks.

Being motivated by “naturalness” and an attempt to generate a natural hierarchy from the Plank scale to the electroweak scale, the models with a warped extra dimension are at their best when the absence of fine tuning is prevalent in all parameters. However, severe constraints from flavour dynamics, especially from the measured



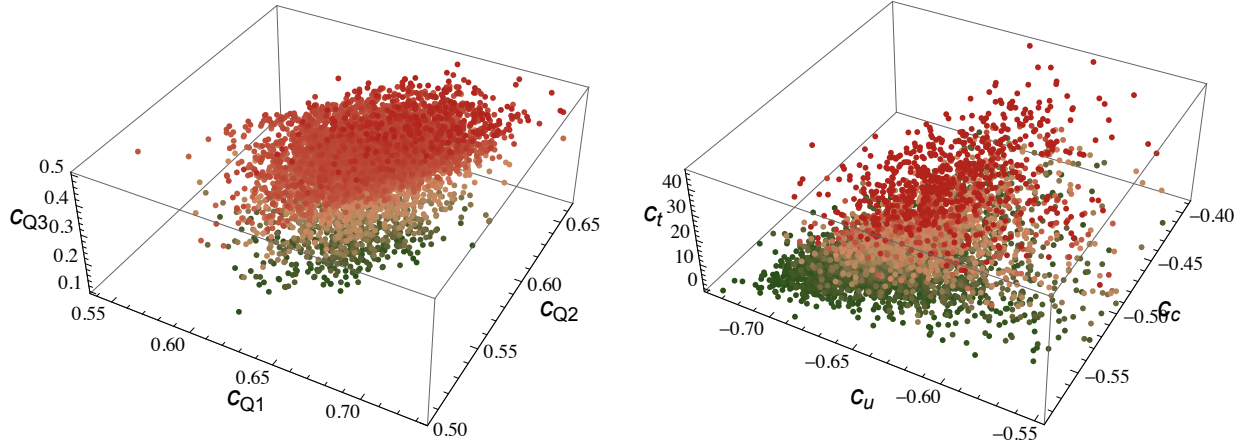


Figure 2: Scatter plot of the bulk mass parameter of the quarks. The figure on the left is for the left handed quark doublets and that on the right is for the right handed  $up$  type quarks.

value of  $\epsilon_K$  force some degree of fine tuning in these models. Even if these models have a custodial symmetry that protects them from electroweak precision constraints, further constraints on the KK mass scale are generated due to constraints from  $\epsilon_K$ . However all this can be alleviated by a judicious choice of the parameter space that we deem as allowed. A “judicious” choice always means the reintroduction of tuning of the parameter sets. However, what is important here is not the presence or absence of tuning, but the degree to which this is necessary. The primary aim of this is to keep the KK mass scale as low as possible to make the model testable in the near future, while not allowing the violation of flavour constraints.

In the light of this argument we shall display the parameter set that has been used in the analysis in a manner that lays it open to naturalness arguments. Amongst, the parameters relevant to this analysis only a subset yield to this argument on naturalness, namely, the Yukawa couplings of the quark sector. We do not discuss the naturalness in the Yukawa couplings of the lepton sector as it is not relevant to this analysis.<sup>6</sup>

The Yukawa coupling of the  $up$  type quarks seem to be quite “anarchic” and random as can be seen from Figure 1, being evenly distributed over the entire parameter space. On the other hand, the  $down$  type Yukawa couplings seem to have a tendency to cluster towards the higher values of the couplings for the first and the second family. It must be kept in mind that the allowed parameter set is shaped by both constraints from electroweak precision tests and flavour constraints, some of which are stronger on the first two families than on the third, especially for the  $down$  type quarks.

In addition to the Yukawa couplings discussed above, it is also informative to take a look at the quark bulk mass parameters, as an interesting pattern is seen when those of the first two families are compared to the third family, Figure 2. For the quark doublet mass parameters  $c_{Q_i}$ , the allowed parameter space is a cone starting from high values of  $c_{Q_i}$ . For the right handed bulk mass parameters  $c_{u,c,t}$ , the cone is less tight and starts from lower values of  $c_{u,c,t}$ . The very large positive values of  $c_t$  should be noted, these are required to generate the right value of the top mass, and thus large overlap between  $t_L$ ,  $t_R$  with the Higgs profile is necessary.

## 5 Rare Decays

In this section we discuss the effects of the class of models introduced above on certain rare decay channels of charm mesons. As should be evident from the previous sections, FCNC from this model of ND comes from tree level exchanges of neutral gauge bosons. The primary contribution on all the decay channels that we have studied, and elucidated on below, comes from the mixing of the new gauge boson states, into  $Z_1$ , the SM neutral massive gauge boson. Contributions that arise from  $Z_2$ ,  $Z_3$  and  $A^{(1)}$  are subdominant or negligible. However, we keep their contributions in the formulation for completion. It also serves the purpose of showing that the addition of the

<sup>6</sup>In this analysis all three families of leptons are all localised with the same bulk mass parameter which also allows one the added benefit of escaping constraints from lepton FCNC.

higher KK modes of the neutral gauge bosons can only produce infinitesimal contributions and careful thought and taking into consideration the nature of the dependence of the contributions on the mass of the gauge states will yield that these contributions steadily decrease in magnitude as we move up the tower and hence fail to introduce considerable enhancements. The corrections from the higher fermion modes in the KK tower can also be similarly argued to be subleading.

We feel that it is not only important to discuss the effects of the warped extra dimension in the rare decays of charm mesons but also correlate it to similar effects in other flavour sectors like beauty and strange. In the following sections we look at correlations between some observables that we derive in this work with observables in the  $\Delta F = 1$  processes in beauty and strange as derived in the works of Blanke et. al. [23]. particularly useful for this approach has been the generosity of the authors of these works in sharing their parameter sets which has allowed us to correlate observables in charm, beauty and strange at exact parameter points over the whole parameter space. The results are indeed quite interesting and will be discussed in the following sections.

### 5.1 $D^0 \rightarrow \gamma\gamma$

Although we do not have explicit calculations of the contributions of ND from the warped extra dimension to  $D^0 \rightarrow \gamma\gamma$ , we feel it is necessary for us to discuss what kind of effects can be expected in this channel for two reasons.

- It is a process in which the models with a warped extra dimension can contribute only at the loop level and have no tree level contribution.
- The SM LD contribution to  $D^0 \rightarrow \mu^+\mu^-$  is driven by the total branching fraction of  $D^0 \rightarrow \gamma\gamma$  [49, 50], and, as pointed out before in [52], is independent of whether the latter is generated by SM or by ND. Hence, enhancements to SM LD contributions to  $D^0 \rightarrow \mu^+\mu^-$  can be generated by ND contributions to the  $D^0 \rightarrow \gamma\gamma$  channel.

Using the notations from [52], the contributions of this model can arise in the 1PR and the 2PR (or 1PI) contributions. The cited work clearly shows that the 1PR contribution dominates the 2PR contribution by about an order of magnitude and ND of the LHT-like framework can hardly enhance the 2PR over the 1PR. A similar argument holds here. Both the 1PR and the 2PR will get contributions only at the loop level and both from the same sources.

- From the mixing of the heavy charged boson states  $W^{1\pm}$  and  $\tilde{W}^\pm$  with the  $W^\pm$  which gives the physical  $W_1^\pm$  states.
- From the heavier physical states  $W_2^\pm$  and  $W_3^\pm$  driving charged currents with SM fermions.

The possibilities of large contribution of ND from the warped extra dimension in  $\Delta C = 1$  processes are solely because FCNC's are generated at the tree level itself. However, for this channel, due to the structure of the contributing diagrams, it is not possible for tree level ND to enhance the rate. The first contributions come only at the loop level as additional gauge states in the loop for the 2PR diagram and enhancements to the effective  $\gamma$  vertex for the 1PR diagram.

Of course as stated earlier and shown in Table 1, the SM LD contribution dominates over the SM SD contribution by three orders of magnitude and if ND has to make its effect felt, through what is essentially SD contributions, it has to enhance the SD rates by more than three orders of magnitude over what the SM can produce and this is not possible for ND of the type being discussed in this paper.

Hence, the verdict is clear.  $D^0 \rightarrow \gamma\gamma$  remains dominated by SM LD contributions within the current framework and hence the SM LD rate for  $D^0 \rightarrow \mu^+\mu^-$  remains controlled by purely SM contributions to  $D^0 \rightarrow \gamma\gamma$  even in the presence of ND from the warped extra dimension.

### 5.2 $D^0 \rightarrow \mu^+\mu^-$

We have seen that the SM SD contributions to this channel are extremely tiny of  $O(10^{-19})$  and the SM LD contribution is almost  $O(10^6)$  larger. If ND has to make its effects felt in this channel through SD contributions then it has to enhance the SM SD rates by more than six orders of magnitude. Figure 3 clearly shows that this is



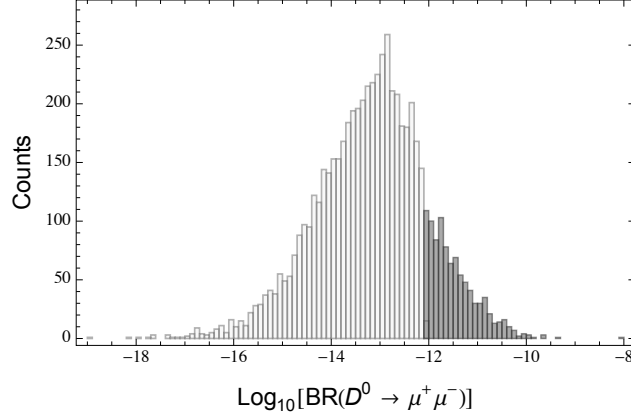


Figure 3: Histogram of the branching fraction of  $D^0 \rightarrow \mu^+ \mu^-$ . The lighter region lies below the estimate of the SM LD contribution. The counts in each bin represents the number of parameter points that contribute to the range of values of  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$  in that bin.

possible in a large part of the parameter space. Enhancements of  $O(10^1) - O(10^2)$  are possible over SM LD rates which is denoted by the darker region of the histogram.

The contributions to this channel are driven by the tree level  $V - A$  contributions to the four fermion vertex  $(\bar{u}c)_{V-A}(\mu^+\mu^-)_{V-A}$  which come primarily from the the mixing of the  $Z^1$  and the  $Z'$  with the  $Z$  yielding the physical  $Z_1$  state. The contributions of the other neutral gauge bosons are negligible to up to a few percent. The new contributions to the  $V - A$  and  $V$  currents in the effective Hamiltonian can be parametrized through the following functions:

$$\begin{aligned}\Delta Y_{V-A}^D &= - \sum_m \frac{g_l^-(m)g_q^-(m)}{4g_{SM}^2 M_m}, \\ \Delta Y_V^D &= - \sum_m \frac{g_l^-(m)[g_q^+(m) - g_q^-(m)]}{4g_{SM}^2 M_m},\end{aligned}\tag{52}$$

where

$$g_{SM} = \left( \frac{G_F \alpha}{\sqrt{8}\pi \sin^2 \theta_W} \right)^{1/2},$$

and the functions  $g_{l,q}^\pm$  parameterize the difference or sum between the left and right handed couplings to the neutral gauge bosons,  $Z\bar{q}_{iL,R}q_{jL,R}$  and  $Zl_{L,R}^+l_{L,R}^-$ , for  $q_i = u, d$  and  $l = \mu$ ; this notation differs from [23], where differences in  $\Delta_{L,R}$  are used to parametrize the  $g_{l,q}^\pm$  functions. These functions are obtained from the effective unitary matrices defined in Equations (50) and (51) for quarks and Equations (45) and (47) for leptons. Thus, the partial decay of the  $D^0$  meson gets the following contribution:

$$\text{BR}(D^0 \rightarrow \mu^+ \mu^-) = \frac{1}{\Gamma_{D^0}} \frac{G_F^2}{\pi} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 f_D^2 m_\mu m_{D^0} \sqrt{1 - 4 \frac{m_\mu^2}{m_{D^0}^2}} Y_{V-A}^D Y_{V-A}^{D*},\tag{53}$$

with  $Y_{V-A}^D$  including the  $Y_0(x), Y_1(x)$  and  $\Delta Y_{V-A}^D$  with the appropriate CKM factors for the first two.

Current searches for this channel are done with the LHCb detector and branching fractions above the SM LD rates can be accessible in the near future. This channel can be searched in super flavour factories too, either in Belle II or in any of the proposed super tau-charm factories. As mentioned earlier, what is important for this channel is that  $D^0 \rightarrow \gamma\gamma$  be measured with at least order of magnitude precision so that the SM LD contribution to this channel can be understood, something that is achievable in BESIII in the near future and is definitely within reach of the super flavour factories. Hence, this is a good channel to look for ND from the warped extra dimension even if no ND effects are seen in  $B_s \rightarrow \mu^+ \mu^-$  as we shall show later in Section 8.

The authors of the reference [77] claim that models with a warped extra dimension cannot leave any effects in  $D^0 \rightarrow \mu^+ \mu^-$  because  $C_7$  has no role to play here. Actually,  $C_{10}$ , which is the only Wilson coefficient that matters in this channel, gets very large contribution from the new degrees of freedom, much larger than what  $C_7$  (or  $C_8$ ) can get and hence  $D^0 \rightarrow \mu^+ \mu^-$  is enhanced significantly.

### 5.3 $D \rightarrow X_u \nu \bar{\nu}$

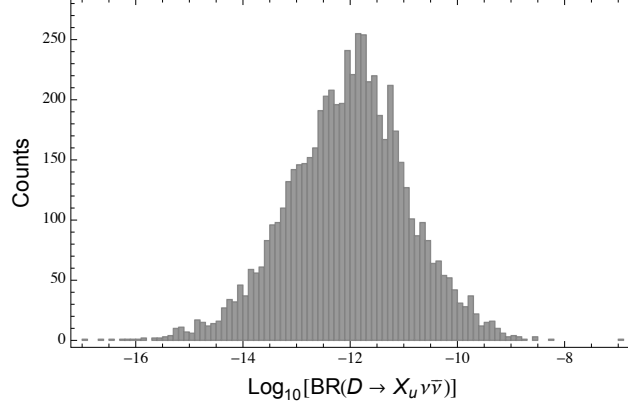


Figure 4: Histogram of the branching fraction of  $D \rightarrow X_u \nu \bar{\nu}$ .

The scenario for  $D \rightarrow X_u \nu \bar{\nu}$  is even more interesting. As we have mentioned before, the SM rate for this channel is driven by SD operators and a detailed analysis with light quark loops is not expected to bring about much enhancements. The new contributions to the  $V-A$  and  $V$  currents of this channel in the effective Hamiltonian can be parametrized through the following functions:

$$\begin{aligned} \Delta X_{V-A}^D &= - \sum_m \frac{[g_l^-(m) + g_l^+(m)]g_q^-(m)}{8g_{SM}^2 M_m}, \\ \Delta X_V^D &= - \sum_m \frac{[g_l^-(m) + g_l^+(m)][g_q^+(m) - g_q^-(m)]}{8g_{SM}^2 M_m}. \end{aligned} \quad (54)$$

Therefore, in this channel, the partial width gets the following contribution:

$$Br(D \rightarrow X_u \nu \bar{\nu}) = \frac{G_F^2 m_D^5}{192\pi^3 \Gamma_D} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \left( \left| X_{V-A}^D + \frac{X_V^D}{2} \right|^2 + \left| \frac{X_V^D}{2} \right|^2 \right), \quad (55)$$

where  $X_{V-A}^D$  includes both the SM and ND contribution and  $X_V^D$  comes from ND only.

It can be seen from Figure 4 that ND from the warped extra dimension can bring about enhancements of  $O(10^5) - O(10^6)$  over the SM rates in a large part of the parameter space. As for the case of the branching fraction of  $D^0 \rightarrow \mu^+ \mu^-$ , this is possible even when ND of such kind can only make negligible or no contributions to  $B_s \rightarrow \mu^+ \mu^-$ .

The source of such enhancement is again the mixing of the ND gauge states with the SM  $Z$  boson producing tree level FCNC. Like in the case of analogous kaons and beauty decays [23], here too there is vector FCNC generated in the ND which is absent in the SM. However, numerically, almost all the enhancement due to ND comes from the  $V-A$  FCNC.

Such large enhancements as we see here can be accessible in super flavour factories. They might be out of reach for the LHCb as the branching fraction is relatively small and the final state comes with two neutrinos. Observation of this channel will provide a very clean and clear signature of ND. If signatures in this channel are found at the higher values of the branching fraction shown in Figure 4, a strong case can be made for tree level enhancement too, as the loop level enhancement seen in [54] is about one or two orders of magnitude larger than the SM estimates.

### 5.4 $D \rightarrow X_u l^+ l^-$

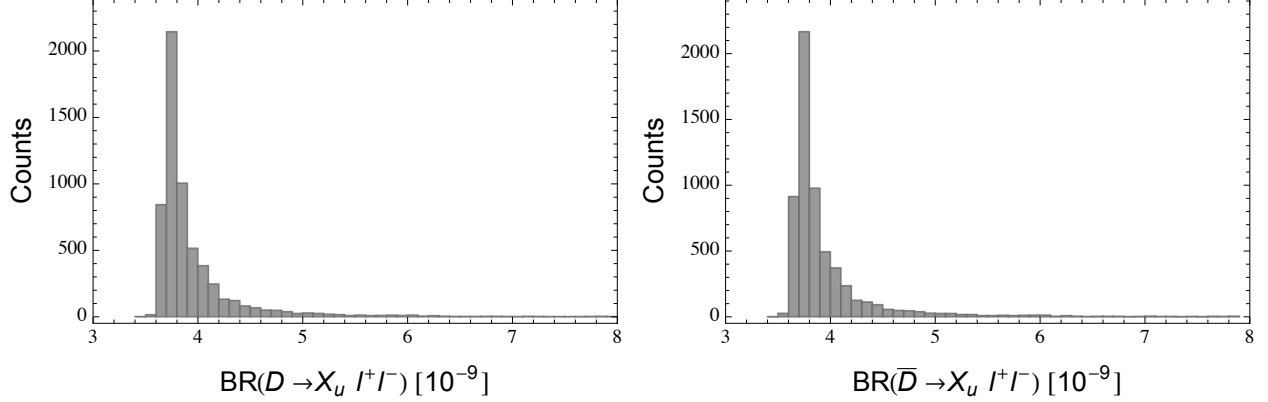


Figure 5: Histogram of the branching fraction of  $D \rightarrow X_u l^+ l^-$ .

The branching fraction of  $D \rightarrow X_u l^+ l^-$  is not expected to have a significant enhancement, as the SM contribution is dominated by the photon penguin contribution where ND can hardly make its effect felt. The other contributions are negligible in comparison to the SM and need to be enhanced by several orders of magnitude in order to be comparable to the SM photon penguin contribution in  $C_9$ . As can be seen in Figure 5, the effects of ND in the branching fraction is quite small,  $O(10\%)$  in most of the parameter space, quite negligible as it is evident from the peak of the distribution lying on the SM value for the branching fraction. The additional contributions to  $C_9$  and  $C_{10}$  from tree-level neutral currents arising from the warped extra dimension are parametrized in the following way:

$$\begin{aligned} \Delta C_9 &= \left[ \frac{\Delta Y_V^D + \Delta Y_{V-A}^D}{\sin^2 \theta_W} - 4 (\Delta Z_V^D + \Delta Z_{V-A}^D) \right], \\ \Delta C_{10} &= -(\Delta Y_V^D - \Delta Y_{V-A}^D), \end{aligned} \quad (56)$$

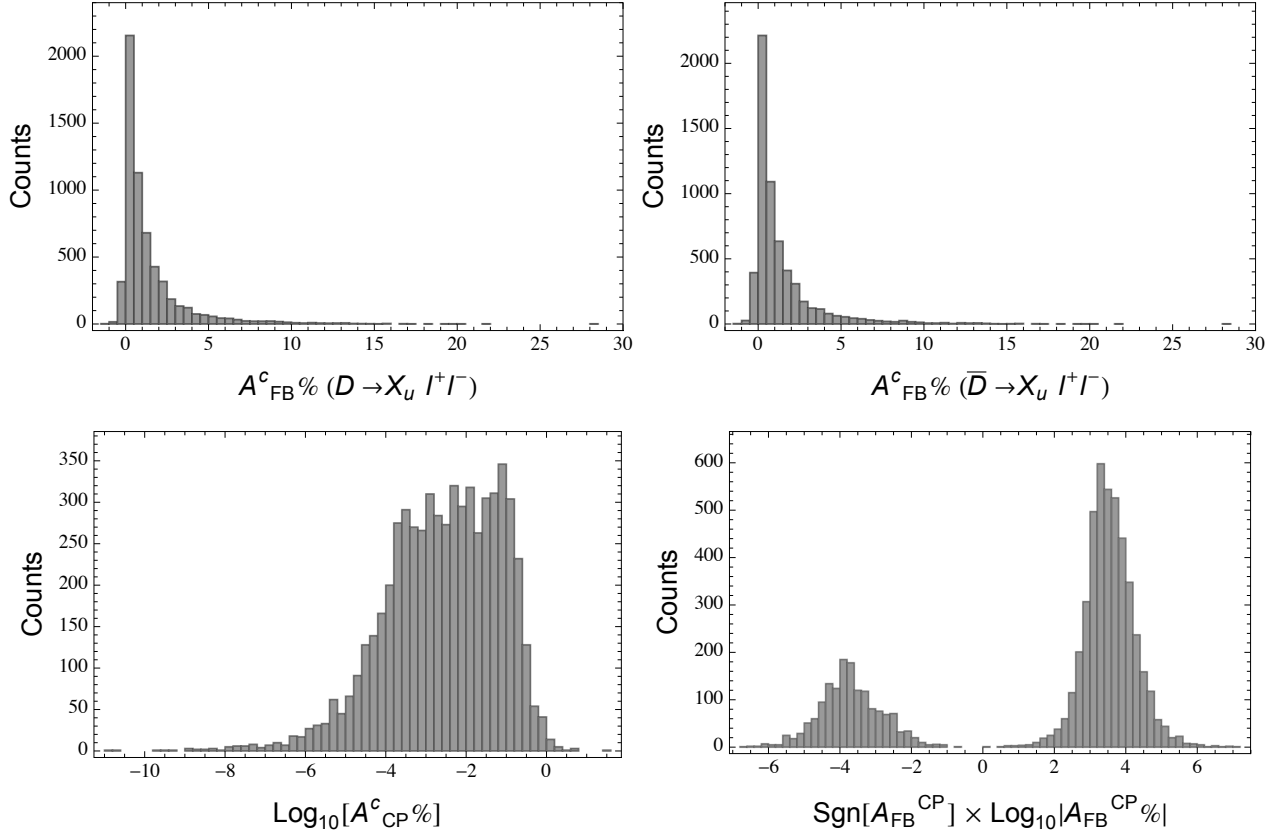
where

$$\Delta Z_{V-A}^D = - \sum_m \frac{[g_l^+(m) - g_l^-(m)] g_q^-(m)}{16 g_{SM}^2 \sin^2 \theta_W M_m}, \quad (57)$$

$$\Delta Z_V^D = - \sum_m \frac{[g_l^+(m) - g_l^-(m)] [g_q^+(m) - g_q^-(m)]}{8 g_{SM}^2 M_m}. \quad (58)$$

While  $C_{10}$  is enhanced by orders of magnitude and  $C_9$  gets significant contributions too, these tree level contributions fail to compete with the photon penguin contribution in  $C_9$  coming from the SM. This is quite like what we saw in the LHT-like models [53] where loop level effects contributed to enhancing  $C_9$ ,  $C_{10}$  and  $C_7(C_8)$ . In addition to this, the branching fraction in these channels are dominated by long distance effects by orders of magnitude. Hence, new dynamics have no chance of showing up in the branching fractions of these channels.

The effects of models with a warped extra dimension in this channel was studied in [77]. They have missed out quite a few subtleties. Firstly, the largest effect that this kind of ND will have on this channel is not through its contribution to the dipole operators in  $c \rightarrow u \gamma$ . In fact the dipole contributions to  $C_7$  and  $C_8$  hardly play a role in determining the size of the branching fraction of  $D \rightarrow X_u l^+ l^-$  even in the SM. It is the photon penguin contribution in  $C_9$  which sets the stage for the SM SD contribution. As we have shown, the photon penguin contribution in  $C_9$  is not enhanced significantly by these models over SM values. The real enhancement is seen in  $C_{10}$  through tree level neutral currents which, however, can at most become comparable to the SM SD photon penguin contribution. Secondly, the branching fraction in this channel is not expected to be enhanced by ND. What is expected to be impacted by the presence of ND are the asymmetries as we show next.

6 Asymmetries in  $D \rightarrow X_u l^+ l^-$ Figure 6: Histogram of the Asymmetries in  $D \rightarrow X_u l^+ l^-$ .

It is a different story altogether for the impact of ND on asymmetries in the  $D \rightarrow X_u l^+ l^-$  channel. We saw earlier that the SM signatures in the asymmetries are extremely tiny. From Figure 6, it is quite obvious that ND intervention in the asymmetries is sizable or even large – i.e., orders of magnitude more than what the SM can produce.

The forward backward asymmetry  $A_{\text{FB}}^c$  can be enhanced to even  $O(5\%)$  in some of the parameter space. This enhancement can be understood from the enhancement of  $C_{10}$  as  $A_{\text{FB}}^c$  depends directly on the magnitude of  $C_{10}$ . Tree level contributions from the mixing of the new gauge states with the SM  $Z$  gauge boson bring about such enhancements and can be seen to be quantitatively more than the loop level enhancement that we had seen in the LHT-like models [53]. It should be noted that in the SM, and in the absence of CP violation (a good assumption within the SM for these channels),  $A_{\text{FB}}^c$  for the conjugate channels should have opposite signs. However, in Figure 6, it can be seen that the asymmetry for both the conjugate channels are of the same sign. This is a clear indication of the existence of CP asymmetry in these channels.

The latter is made clear by the up to  $O(1\%)$  CP asymmetry,  $A_{\text{CP}}^c$ , that can be seen from the graph in Figure 6. Although CP asymmetry remains small for quite an insignificant fraction of the parameter space, there are possibilities of measurable CP asymmetry in a significant fraction of the parameter space too. This is not surprising as there are new phases coming from the new mass mixing matrices of the fermions affecting the neutral currents driven by new gauge states.

As expected from large  $A_{\text{FB}}^c$  and sizeable  $A_{\text{CP}}^c$ , the  $A_{\text{FB}}^{\text{CP}}$  is enhanced by orders of magnitude in this scenario of ND. As seen in the LHT-like models [53], this asymmetry can be as high as  $O(10^4\%)$  and can be both positive and negative although it does show a tendency of being positive in a lot more of the parameter space. This can

be attributed to the prevalence of positive  $A_{\text{FB}}^c$  over negative ones in a large part of the parameter space, but can also arise from  $A_{\text{FB}}^c$  being larger in  $D$  than in  $\bar{D}$  in most parts of the parameter space<sup>7</sup>.

All three asymmetries provide unambiguous signatures of ND as SM values for these are very negligible. Probing these asymmetries should be on any menu that includes the experimental study of rare charm decays. To probe these asymmetries experiments have to be sensitive to at least the SD contribution to the branching fraction. While it is not necessary to discern the LD from the SD dynamics in the branching fraction itself unless ND changes the shape of the distribution of the differential branching fraction, this reach is necessary as the asymmetries are generated by short distance dynamics primarily. While LHCb might be sensitive to such small branching fractions, it might be complicated to extract the asymmetries from crowded background. The cleaner environment and full coverage of super flavour factories would be much better for such measurements.

## 7 Correlations between different $\Delta C = 1$ observables

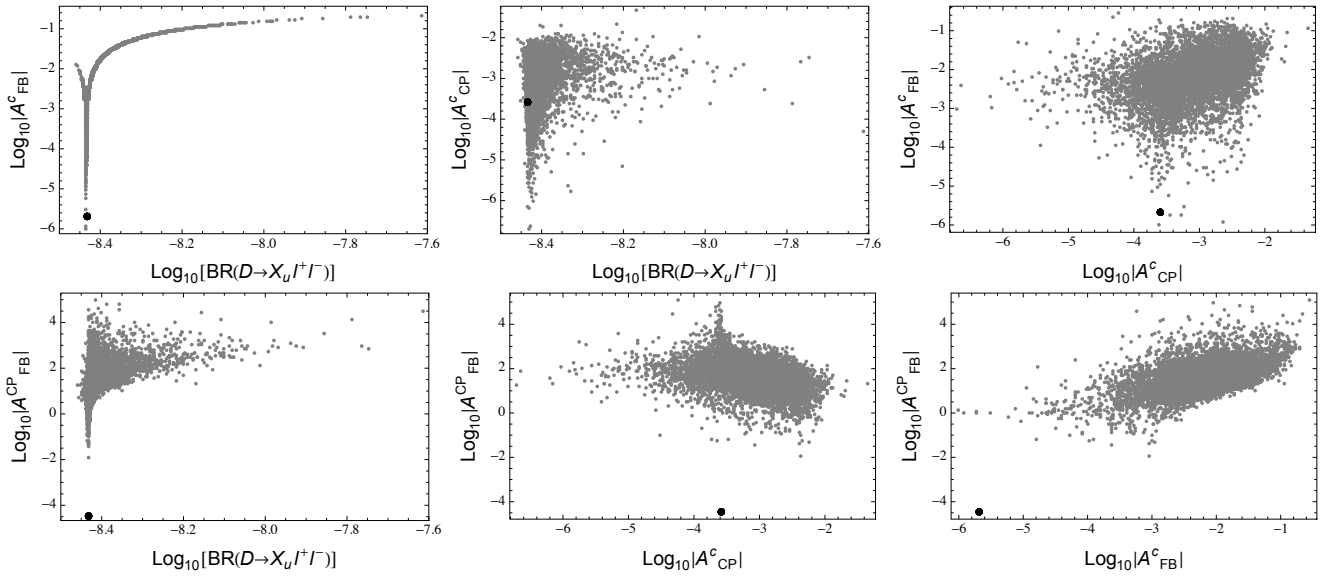


Figure 7: Correlations between  $D \rightarrow X_u l^+ l^-$  observables. The black dot in every graph represents the SM contributions to the corresponding observables.

Having studied the observables themselves, it is instructive to take a look at the correlation between these different observables too, for three reasons:

- Enhancement of one observable is not always accompanied by the enhancement of another, *even in the same decay mode*. Hence correlations tell us a lot about what sets of observables to look for given a subset of the parameter space.
- Correlations give us an insight into the underlying theoretical framework, in this case, the Wilson coefficients and their relative enhancements and dominance. Generalisation can follow from such an overview.
- Most importantly, it is necessary to look into correlations between observables in different flavour sectors. This serves not only as a probe of flavour non-universality but also gives directions in search strategies in experiments.

The first two we shall address in this section while the last case we shall study in the next section.

<sup>7</sup>This can be concluded only since a very tiny part of the parameter space has  $A_{\text{FB}}^{\text{CP}}$  less than 1%

Amongst the decays studied in this work, the three body inclusive mode  $D \rightarrow X_u l^+ l^-$  is the richest in contributions from different operators. Studying the correlations between the branching fraction and the asymmetries in this mode gives us an insight into how ND contributes to Wilson coefficients  $C_7(C_8)$ ,  $C_9$  and  $C_{10}$ . It is important to note here that the ND scenario under consideration in this work has new sources of FCNC at tree level only. Hence, significant contributions cannot be made to photon or gluon penguins unlike the case of LHT-like models [53]. The only significant contributions that are made are to  $C_9$  and  $C_{10}$ .  $C_9$  is dominated by the SM photon penguin as mentioned earlier and hence the only Wilson coefficient that gets a significant enhancement over its SM value is  $C_{10}$ . However, new phases can (and do) appear in both  $C_9$  and  $C_{10}$  due to the new fermion mixing matrices.

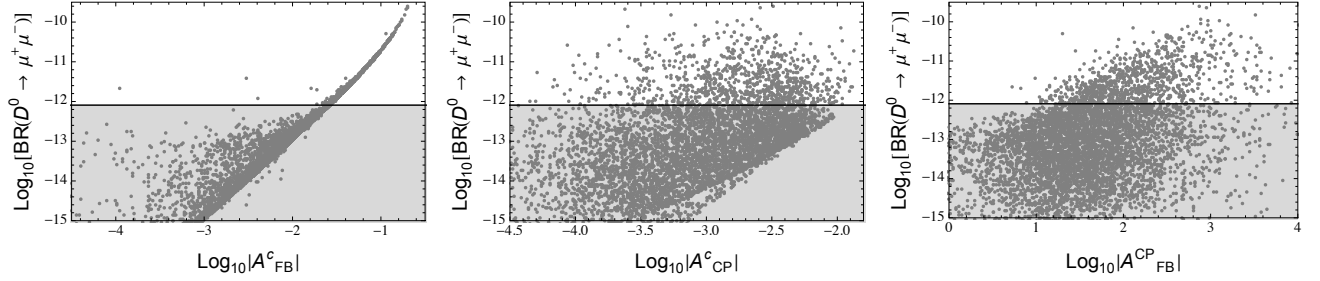


Figure 8: Correlations between asymmetries in  $D \rightarrow X_u l^+ l^-$  and  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$ . The region below the black line shaded in grey represent the part of the observable space where ND contribution to  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$  is subdominant to SM LD contribution to the same.

It is clear from the tight correlation, in Figure 7, between the branching fraction in  $D \rightarrow X_u l^+ l^-$  and the  $A_{\text{FB}}^c$  that the enhancements in both come from an enhancement of  $C_{10}$  as the latter is linearly dependent on  $C_{10}$ . As mentioned before,  $C_9$  is already large in SM and does not get enhanced much by ND. The correlations between  $A_{\text{CP}}^c$  and the other variables are very loose as the former is sensitive to the phases in  $C_9$  and  $C_{10}$ . The only other set of observables that seem to be correlated are the  $A_{\text{FB}}^c$  and  $A_{\text{FB}}^{\text{CP}}$  where one seems to be increasing with the other. The former is sensitive to the magnitude of  $C_{10}$  while the latter is sensitive to the phase in  $C_{10}$  which points at significant enhancements to both the magnitude and the phase in  $C_{10}$  from ND.

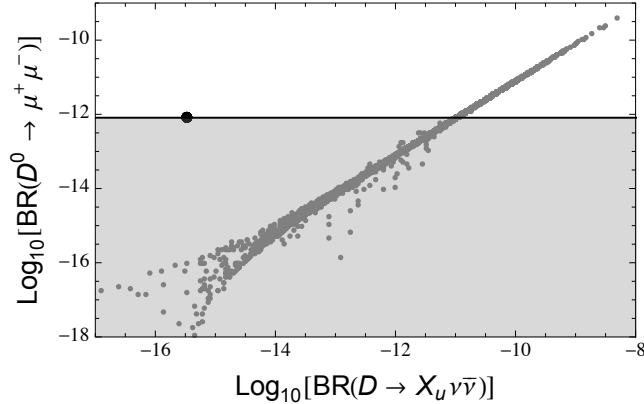


Figure 9: Correlations between  $\text{BR}(D \rightarrow X_u \nu \bar{\nu})$  and  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$ . The black dot represents the SM estimate of  $\text{BR}(D \rightarrow X_u \nu \bar{\nu})$ . The grey region represents the part of the parameter space where only  $\text{BR}(D \rightarrow X_u \nu \bar{\nu})$  gets enhanced above SM estimated while ND in  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$  remains subdominant to SM.

In Figure 8 we look at the correlation between the asymmetries in  $D \rightarrow X_u l^+ l^-$  and  $D^0 \rightarrow \mu^+ \mu^-$ . The grey band denotes the part of the parameter set that fails to overcome the SM LD contribution to  $D^0 \rightarrow \mu^+ \mu^-$ . Therefore the points in the white region of each of the graphs represent enhancements to *both* the branching fraction

of  $D^0 \rightarrow \mu^+\mu^-$  and the asymmetries in  $D \rightarrow X_u l^+ l^-$  that, if observed, would hint at the presence of ND. All the asymmetries show possibilities of enhancements to large values while the branching ratio of  $D^0 \rightarrow \mu^+\mu^-$  remains large and distinguishable from SM LD contributions. The only two pairs of observables that are clearly correlated are the branching fraction of  $D^0 \rightarrow \mu^+\mu^-$  and  $A_{\text{FB}}^c$  since both depend purely on the size of  $C_{10}$ .

The case for the correlations between the branching fractions of  $D^0 \rightarrow \mu^+\mu^-$  and  $D \rightarrow X_u \nu \bar{\nu}$  is shown in the plot on the right in Figure 9. In the grey region the branching fraction of  $D^0 \rightarrow \mu^+\mu^-$  lies shrouded in SM long distance dynamics. We see here that there are significant parts of the parameter space in which simultaneous enhancements to the branching fraction of both  $D^0 \rightarrow \mu^+\mu^-$  and  $D \rightarrow X_u \nu \bar{\nu}$  are possible due to the tight correlation between the two observables. It can also be seen that even if ND shows up in  $D \rightarrow X_u \nu \bar{\nu}$ , it can fail to overcome the SM contribution to  $D^0 \rightarrow \mu^+\mu^-$  in a large part of the parameter space. However, it should be kept in mind that in the latter regime, it will be quite difficult for experiments to measure the branching fraction of  $D \rightarrow X_u \nu \bar{\nu}$  due to its small size.

## 8 Correlations between strange, charm and beauty

It has been long known that the intervention of ND in the different flavour sectors can be different. Firstly, there are possibilities of non-universal scenarios of flavour dynamics that can manifest themselves and leave different signatures in different flavour sectors. Secondly, manifestations of ND also need to dominate over SM contributions to flavour observables to be seen in flavour dynamics. While this is not possible in some flavour observable, some others, specially from charm dynamics, are ripe for such occurrences. A correlation study between  $\Delta S = 1$  and  $\Delta B = 1$  observables was done in [23]. It is quite instructive to also compare flavour dynamics in two different sectors.

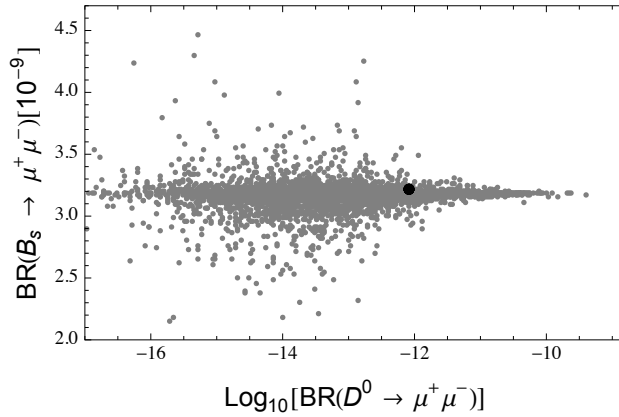


Figure 10: Correlations between  $\text{BR}(B_s \rightarrow \mu^+\mu^-)$  and  $\text{BR}(D^0 \rightarrow \mu^+\mu^-)$ . The black dot represents the SM LD estimate of  $\text{BR}(D^0 \rightarrow \mu^+\mu^-)$ . The y axis is well within the 95% C.L. values for the current LHCb results for  $\text{BR}(B_s \rightarrow \mu^+\mu^-)$  [32].

In Figure 10 we show two observable and their correlations. The plot shows the correlation between  $\text{BR}(B_s \rightarrow \mu^+\mu^-)$  and  $\text{BR}(D^0 \rightarrow \mu^+\mu^-)$ . The values on the y axis lie well within the 95% C.L. values from the current experimental measurement of the branching fraction of  $B_s \rightarrow \mu^+\mu^-$  announced recently by LHCb [32].

$$1.1 \times 10^{-9} < \text{BR}(D^0 \rightarrow \mu^+\mu^-) < 6.4 \times 10^{-9} \quad \text{at 95\% C.L.} \quad (59)$$

There is not much correlation between the two observables. However, it is clear that even if the experimental errors are reduced and the branching fraction of  $B_s \rightarrow \mu^+\mu^-$  is very close to the SM expectation and ND effects are indiscernible in that channel, large enhancements can show up in the branching fraction of  $D^0 \rightarrow \mu^+\mu^-$  and even dominate over the SM LD contributions to the same. This stems from the fact that SM contributions to  $B_s \rightarrow \mu^+\mu^-$  (like in most of beauty dynamics) are quite large. Hence, ND can hardly make orders of magnitude enhancement there. The enhancements to  $B_s \rightarrow \mu^+\mu^-$  can clearly be seen here to be of  $O(10\%)$  while in the

same parameter space it is orders of magnitude for charm. Hence, new dynamics can manifest itself in charm while keeping a low profile in beauty and vice versa, although the latter is a more difficult situation to deal with experimentally. We advocate the study of ND in charm lest nature choses the former.

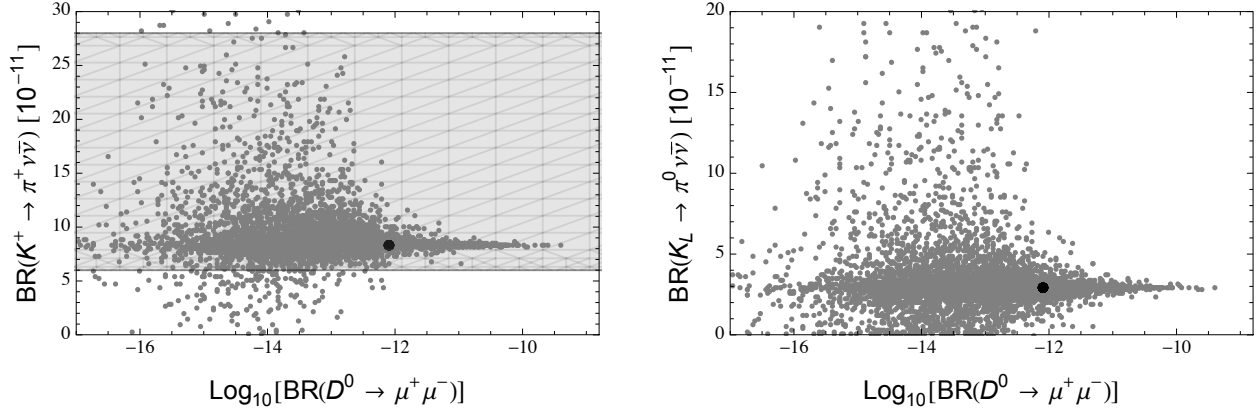


Figure 11: Correlations between  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ ,  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$ . The black dot represents the SM LD estimate of  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$  along with the SM estimates for  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ . The grey band on the left plot is the  $1\sigma$  experimental bounds on  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ .

As mentioned earlier, a comparison between FCNC in strange and beauty dynamics was done in [23, 24]. It was shown that the observables in the strange and beauty sectors are anti-correlated and can show enhancements in complementary parts of the parameter space. However, even in the  $\Delta S = 1$  observables studied in the koan sector, namely the branching fractions of  $K_L \rightarrow \mu^+ \mu^-$ ,  $K_L^{(+)} \rightarrow \pi^{0(+)} \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 l^+ l^-$  the enhancements are modest and of at most an order of magnitude. The last two are compared in Figure 11 with  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$ . The experimental limits for these two koan decay modes are [89, 90]:

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.7 \pm 1.1 \times 10^{-10} \quad (60)$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 2.6 \times 10^{-8} \text{ at } 90\% \text{ CL} \quad (61)$$

to be compared to the SM values of [91]

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 7.83 \pm 0.82 \times 10^{-11} \quad (62)$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.49 \pm 0.39 \times 10^{-11} \quad (63)$$

While it is true that if the experimental value stays close to the central value for  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ , this kind of ND cannot make large enhancements to  $\text{BR}(D^0 \rightarrow \mu^+ \mu^-)$  it is also true in some parts of the parameter space, even if ND leaves very small signatures in the former, the latter can be enhanced by orders of magnitude in a large part of the parameter space. With experimental limits being quite close to the SM, the case of ND suffers the same fate as in beauty: there is a possibility of enhancements for ND but it lies shrouded in the shadows of the SM. We emphasise again, the case of charm is very different. Where ND can only leave modest effects in strange and beauty, it can leave a severe impact in charm, sometimes orders of magnitude beyond the reach of the SM.

## 9 Analysis of the parameter space

While we see enhancements in quite a few observables in rare charm decays, it might be instrumental to ask at this point whether such enhancements lead us to a preference for any particular part of the parameter space for the generation of observable new dynamics. In Section 4 we displayed the distribution of the Yukawa couplings



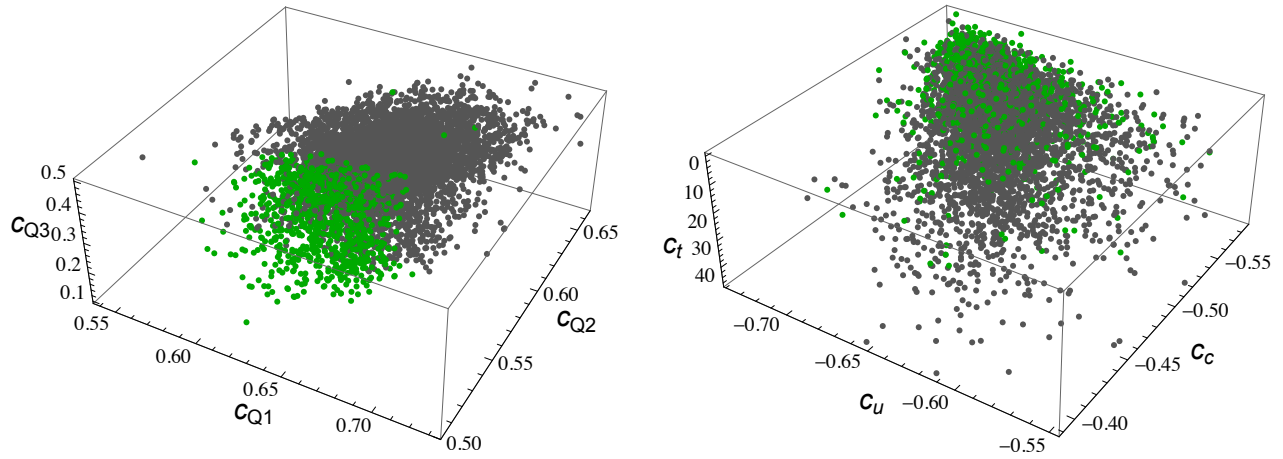


Figure 12: The bulk mass parameters of the three families of quarks and their correlation to enhancements in  $BR(D^0 \rightarrow \mu^+\mu^-)$ . The figure on the left shows the bulk mass parameters for the left handed quark doublets and the one on the right shows those for the right handed singlet quarks. The green (lighter) points represent the part of the parameter space that can generate enhancements to  $BR(D^0 \rightarrow \mu^+\mu^-)$  beyond the SM LD estimates.

and the bulk mass parameter that was used in this analysis. In Figure 12 we take another look at the space of the bulk mass parameters in the light of ND contributions to  $BR(D^0 \rightarrow \mu^+\mu^-)$  beyond the SM LD estimates.

The green (lighter) points represent the part of the parameter space in which we see enhancements to  $BR(D^0 \rightarrow \mu^+\mu^-)$  beyond the SM LD estimates. We clearly see, from the figure on the left, that there is a strong preference for the bulk mass parameter of the second family of left handed SM quark doublets to be at lower values close to 0.5 when such enhancements occur, while there is no clear preferences for the values of the other two quark doublet bulk mass parameters. The figure on the right also shows that such enhancements prefers lower values of  $c_t$ , the bulk mass parameter of the right handed top quark.

The Yukawa couplings do not show any trends and the parameter points generating enhancements in  $BR(D^0 \rightarrow \mu^+\mu^-)$  beyond the SM LD estimates are more or less randomly distributed in over the entire parameter space.

## 10 Inference

In this analysis we have looked at a particular flavour structure of possible ND from a model of warped extra dimension and tried to see what signatures it can leave in the dynamics of the charm quark. However, this analysis goes beyond just examining the latter effects in one particular model. In essence, this analysis can well achieve some model independence if we look into the details of the ND signatures left in the observables that we have studied.

Let us first take a look at the flavour structure we have used. The model essentially has a set of new gauge bosons, not only from new gauge sectors but also from the appearance of KK towers. While it has both new neutral gauge bosons and charged gauge bosons, the former is more significant in an analysis of FCNC as such is generated at tree level itself, which is a novel signature of the ND absent in the SM. So it is sufficient to consider only the neutral gauge bosons in the analysis of FCNC.

Although the fermion sector also has new degrees of freedom, none of that contribute to FCNC at any degree of significance as the tree level diagrams with SM fermions and new gauge bosons are dominant by far. While the actual size of the FCNC depends on multiple factors like the overlap of the fermionic profiles with the gauge boson profiles, the localisation of the Higgs, the Yukawa parameters and the KK mass scale, many of these are not independently scalable as there are multiple experimental constraints that tie these together. We shall go ahead with some generalisations of this flavour structure and its signatures in charm changing neutral currents (CCNC) based on two arguments, firstly, the nature of the spectrum of the degrees of freedom and secondly, the KK mass

scale.

The localisation of the gauge bosons are fixed once the KK mass scale is fixed along with the boundary conditions to match up with the SM gauge content and, in the case of the current model, a wish to preserve custodial symmetry in the IR brane. We shall not consider the localisation of the Higgs as small changes in the same within allowed model constraints do not have any large effects in the observables studied<sup>8</sup>. The localisation of the fermions can change within certain limits as can the Yukawa parameters. The last two parameter sets are what determine the size of CCNC, the KK mass scale remaining the same.

From the above arguments, we can define a generic flavour structure:

- New neutral and charged gauge boson states.
- Overlap of the SM fermions with the new gauge boson states generating tree level FCNC.
- New angles and phases coming from fermion mixing matrices producing flavour non-diagonal neutral currents.<sup>9</sup>

We claim that as long as models with a warped extra dimension are built of this flavour structure it will contribute to CCNC significantly. Inclusion of more neutral gauge fields can only enhance the current effects seen but not by orders of magnitude as long as such inclusion follows all the requisite constraints from electroweak precision tests and other precisely measured flavour observable. Here we have considered a non-ad-hoc flavour structure that is not “tuned” to give effects in the *up* type quarks. Moreover, we have used a parameter set whose effects have already been studied in [23, 24]. Hence it gives us a comprehensive picture of the effects of these kinds of flavour structures in the dynamics of *both up* type and *down* type quarks.

Our second argument brings us to consider the limitations of the LHC in direct searches. One motivation for keeping ND at the TeV scale is to make sure that we see it at the LHC direct searches. In many models with a warped extra dimension it is possible to keep low KK scales using different theoretical technologies. Yet, one is led to wonder what will happen if we fail to discover these low KK scales, i.e., nature has “heavier” plans for us.

The most simplistic way one can argue the KK scale dependence in the observables we have studied is to state that the amplitudes depend roughly on the inverse square of the KK mass scale and hence the branching fractions and the asymmetries depend on the same to the fourth power. The KK mass scale that we have used is about 2.45 TeV. It would be quite justifiable to say that if the KK scale really lies at around 5 TeV or more<sup>10</sup>, direct searches at LHC will start having a problem seeing new degrees of freedom. Yet the effects in charm dynamics would at best be lowered by an order of magnitude. Considering what the numbers tell us, charm dynamics would still be in the game for showing ND effects even if direct searches at LHC and possibly, both beauty and kaon dynamics would be out of the game within the ambits of such a ND scenario. Suffice it to say that charm has been both cursed and blessed by tiny SM signatures.

## 11 Summary

The possibilities of the existence of ND coming from a warped extra dimension is an intriguing solution to many problems and questions that face us today. Yet these models are severely constrained by what has already been measured. While efforts are being made to both see the new degrees of freedom in direct searches and probe new dynamics in flavour dynamics, we put in our share of trying to see the impacts of these structures on the dynamics of the charm quark and their features.

We also, in general, strongly disagree with the conclusion in [77] that the “only” place the models with a warped extra dimension can show its effect are in CP asymmetries in  $D \rightarrow X_u \gamma$  besides  $\Delta A_{CP}$ <sup>11</sup>. We think we have convincingly argued in this paper that models of this kind can have large effects well beyond the SM estimates in multiple rare decays including  $D^0 \rightarrow \mu^+ \mu^-$ ,  $D \rightarrow X_u \nu \bar{\nu}$  and asymmetries in  $D \rightarrow X_u l^+ l^-$  which can be within experimental reach in the near future. Also, we propose a study of higher multiplicity hadronic decays of the charmed meson both experimentally and theoretically, although a lot of tools need to be developed for these.

In this work we have shown that:

<sup>8</sup>For a study of the dependence of CP violation in  $D^0 \rightarrow \pi^+ \pi^- / K^+ K^-$  on the Higgs localisation cf. [77]. The fact that the Higgs localisation does not matter much in the effects that we see in our work shows that these effects are independent of what needs to be done to generate  $\Delta A_{CP}$ .

<sup>9</sup>In the absence of any new angles and phases, the model reduces to the MFV kind which we do not study.

<sup>10</sup>We are assuming the reality of the existence of a warped extra dimension.

<sup>11</sup>For detailed studies of the connection between these two observables cf. [92, 93].

- These kind of flavour structure can leave large effects in charm, sometimes orders of magnitude larger than what the SM can generate.
- While these models leave moderate effects in beauty and strange dynamics, the effects in charm need not be moderate. This can be achieved even without giving the  $up$  type quark sector a special dynamical advantage.
- The effects in charm dynamics are not tightly correlated with those in beauty and strange, i.e., even when this kind of ND can leave negligible contributions to both beauty and strange dynamics, it can leave very large contributions to charm dynamics.
- These tree level effects coming from this class of models can be larger than the loop-level enhancement that we saw in the LHT-like models for the same observables.

An important part of this work is the study of correlations between the dynamics of the *down* type quark sector, i.e.,  $B$  and  $K$  decays and the dynamics of an  $up$  type quark in  $D$  decays. Not only do we connect dynamics of different quarks through such an analysis, but we also connect the two sectors of quarks within a parameter space of new dynamics that obeys experimental constraints both from EWPD and flavour observables. We show that even while ND might not manifest itself prominently in the dynamics of the *down* type quarks, it can well bring about orders of magnitude enhancements to dynamics of the charm meson as we have highlighted above. It should be noted that the class of model we have studied here is not motivated to produce effects in any of the flavour sectors, collectively or singly. In spite of this, we not only see possible large impacts in flavour dynamics, but also a clear distinction between effects in different flavour sectors.

While it continues to be true that accessing rare charm dynamics is statistically challenging and theoretically not well understood even at the level of the SM contributions, we have made some strong cases for ND intervention way beyond what the SM can generate. These effects not only lead to the observables coming within reach of current experiments and future super flavour factories, but also sidestep the problem of determining SM contributions theoretically due to their large size.

Finally, we want to state that indirect evidences for ND are based on flavour dynamics beyond the SM in general. However we do not like to go to a shopping mall to find anything that is just beyond the SM flavour dynamics; we greatly prefer to think about flavour dynamics that originates from a motivation to find solutions to the hierarchy problem or challenge of the SM. Previously we have worked about LHT to deal with models of one class, now we have done the same for the Warped Extra Dimension, *a la*, the class of Randall-Sundrum models. Now we shall wait and hope that nature conspires on our side.

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